Power train :-

Torque at clutch:

$$T_c = T_e - I_e \alpha_e \quad —(1)$$

where,
T_e = engine torque
I_e = engine rotational inertia.
α_e = Engine rotational acceleration.

Torque at the driveline or output of transmission box.

$$T_d = [T_c - I_t \cdot \alpha_e] N_t \quad —(2)$$

where,
$I_t \Rightarrow$ Inertia of transmission.
N_t = Gear ratio of transmission

$$N_t = \frac{T_d}{T_c} = \frac{\omega_c}{\omega_d} = \frac{\alpha_c}{\alpha_d}$$

Torque at the axle,

$$T_a = [T_d - I_d \cdot \alpha_d] \cdot N_f \quad —(3)$$

or

$$T_a = F_x \cdot r + I_w \cdot \alpha_w \quad —(4).$$

where
N_f = Gear ratio of final drive

$$N_f = \frac{T_{aw}}{T_d} = \frac{\omega_d}{\omega_{aw}} = \frac{\alpha_d}{\alpha_{aw}}$$

$$\therefore N_t = \frac{\alpha_e}{\alpha_d} \quad —(5)$$

$$N_f = \frac{\alpha_d}{\alpha_w} \quad —(6)$$

So, now taking equtn (4)

$$T_a = F_x \cdot r + I_w \cdot \alpha_w$$

$$F_x \cdot r = T_a - I_w \cdot \alpha_w$$

$$F_x = \frac{1}{r}[T_a - I_w \cdot \alpha_w]$$

$$F_x = \frac{1}{r}[\{T_d - I_d \cdot \alpha_d\} \cdot N_f - I_w \cdot \alpha_w]$$ Putting equtn (3)

$$F_x = \frac{1}{r}[T_d \cdot N_f - I_d \cdot \alpha_d \cdot N_f - I_w \cdot \alpha_w]$$

$$= \frac{1}{r}[[T_c - I_t \cdot \alpha_e] N_t \cdot N_f - I_d \cdot \alpha_d \cdot N_f - I_w \alpha_w]$$ Putting equtn (2)

$$= \frac{1}{r}[T_c N_{tf} - I_t \cdot \alpha_e N_{tf} - I_d \cdot \alpha_d \cdot N_f - I_w \alpha_w]$$

Putting equation (1),

$$F_x = \frac{1}{r}\left[\{T_e - I_e\alpha_e\} N_{tf} - I_t \alpha_e N_{tf} - I_d \alpha_d N_f - I_w \alpha_w\right]$$

$$= \frac{1}{r}\left[T_e N_{tf} - I_e \alpha_e N_{tf} - I_t \cdot \alpha_e N_{tf} - I_d \alpha_d N_f - I_w \alpha_w\right]$$

∴ from equation (5) & (6)

$$\alpha_e = N_t \cdot \alpha_d$$

$$= N_t \cdot N_f \cdot \alpha_w$$

$$\boxed{\alpha_e = N_{tf}\, \alpha_w}$$

So putting in above equation

$$F_x = \frac{1}{r}\left[T_e \cdot N_{tf} - I_e N_{tf}^2 \cdot \alpha_w - I_t N_{tf}^2 \alpha_w - I_d \alpha_d N_f - I_w \alpha_w\right]$$

from equation (8).

$$F_x = \frac{1}{r}\left[T_e N_{tf} - \{(I_e + I_t) N_{tf}^2 \alpha_w + I_d N_f^2 \alpha_w + I_w \alpha_w\}\right]$$

$$F_x = \frac{1}{r}\left[T_e N_{tf} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \alpha_w\right]$$

$$F_x = \frac{T_e N_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{\alpha_w}{r}$$

~~from equation~~ ∴ $\alpha_w = \dfrac{a_x}{r}$

$$F_x = \frac{T_e N_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2}$$

The effect of mechanical & viscous losses in driveline component is added by taking into ~~the~~ account the inefficiencies of driveline component (transmission box, ~~differential~~ differential, axles, driveshaft

The eqn becomes, $\eta_{tf} \rightarrow$ efficiency of driveline com.

$$F_x = \underbrace{\frac{T_e N_{tf} \eta_{tf}}{r}}_{} - \underbrace{\left[(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w \right] \frac{a_x}{r^2}}_{}$$

Steady state Tractive force available at ground to overcome road load forces and accelerate the vehicle.

loss of tractive force due to Inertia of engine & drive Train components.

The term in bracket represent equivalent inertia of each component amplified by square of Gear ratio.

The Forces acting on vehicle are,

$$Ma_x = \frac{W}{g} a_x = F_x - R_x - D_A - R_{hx} - W\sin\theta$$

F_x includes engine torque and Inertia torque.

or

we can just rotational inertia can be lumped with mass of vehicle.

$$\boxed{(M + M_r)\, a_x = \left(\frac{W + W_r}{g}\right) a_x = \frac{T_e N_{tf} \eta_{tf}}{r} - R_x - D_A - R_{hx} - W\sin\theta}$$

where,

M_r = Equivalent mass of rotating components.

The combination of two masses is called as

"effective mass" = $M + M_r$.

"mass factor" = $\frac{M + M_r}{M}$

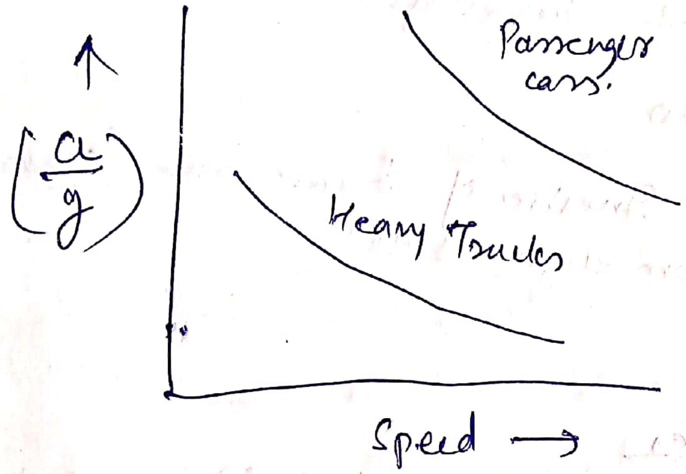
The value of mass factor = $1 + 0.04 + 0.0025\, N_{tf}^2$

so from equ^n (2)

we can say, that

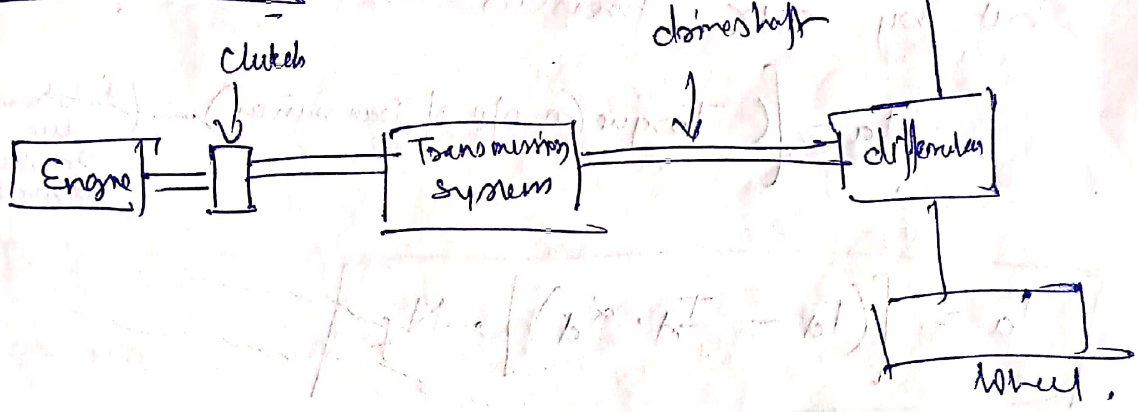
acceleration characteristic decrease with increase in speed

i.e $a_x \propto \frac{1}{V}$



acceleration of heavy trucks is less due to less Power to weight ratio.

(B) Power train



- Engine torque is reduced by an amount of inertia of rotating Components.
- Torque delivered by clutch to transmission system,

clutch torque = (engine torque) − (~~Inertia of engine to produce~~ Torque due to inertia)

$$\boxed{T_c = T_e - [I_e \alpha_e]}$$

T_c = Torque @ clutch

T_e = Engine torque @ given speed

I_e = Engine rotational inertia. α_e = accl^n of engine.

— Torque at o/p of transmission is amplified by gear ratio, but also decreased by inertia of gears & shafts.

~~Ro~~ o/p torque,

$$\boxed{T_d = (T_c - I_t \cdot \alpha_e) \cdot N_t}$$

T_d = o/p torque at driveshaft

N_t = Gear Ratio

I_t = Rotational inertia of transmission (seen from engine side)

— Torque ~~to~~ the axles

Torque at axle = (Tractive force + Inertia of wheels)

$$\boxed{T_a = F_x r + I_w \cdot \alpha_w}$$

F_x = Tractive force at ground
r = Radius of wheel
I_w = Inertia of wheel
α_w = Ang. accln of wheel

but by our previous expressions

$$T_a = \left[(\text{Torque @ o/p of Transmission}) - \left(\begin{array}{c} \text{Rotational Torque} \\ \text{due to} \\ \text{Inertia of} \\ \text{drive line} \end{array} \right) \right]$$

$$\boxed{T_a = \left[(T_d - I_d \cdot \alpha_d) \right] \cdot N_f}$$

where

T_d = Torque at driveshaft

I_d = Inertia of ~~driveline~~ drive shaft

α_d = Rotational acceleration of driveshaft.

N_f = ~~Num. Gear~~ velocity ratio of final drive.

but

Speed ratio of final drive,

$$\boxed{N_f = \frac{\alpha_d}{\alpha_w}} = \frac{\text{i/p acceleration}}{\text{o/p acceleration}}.$$

Transmission drive

$$\boxed{N_t = \frac{\alpha_e}{\alpha_d}}$$

So we can say that

$$\boxed{\alpha_e = N_t \cdot (N_f \cdot \alpha_w)}$$

we know from the tractive effort condition for adhesion,

$$F = \frac{T_w}{r} = \frac{T_w \cdot N_{tf} \cdot \eta}{r}.$$

Tractive effort;

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r}.$$

But this tractive effort will have certain losses hence,

$$F_x = \frac{T_e N_{tf} \cdot \eta_{tf}}{r} - \frac{1}{r}\{\text{losses due to Inertia Torque}\}$$

$$F_x = \frac{T_e N_{tf} \cdot \eta_{tf}}{r} - \frac{1}{r}\{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\}\alpha$$

$$F_x = \frac{T_e N_{tf} \cdot \eta_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\}\frac{a_x}{r^2}$$

So, the eqn becomes

$$\boxed{F_x = \frac{T_e N_{tf} \cdot \eta_{tf}}{r} - \left\{ (I_e + I_t) \cdot N_{tf}^2 + I_d N_f^2 + I_w \right\} \frac{a_x}{r^2}}$$

~~① First term R.H.S. represent~~
where,

T_e = engine torque

N_{tf} = Combined Gear Ratio of Transmission & final drive

η_{tf} = Combine efficiency of — " — " — .

① First term of R.H.S represents, steady state tractive force available at ground to overcome road load forces of aerodynamic & rolling resistance to accelerate or climb a grade.

② Second term represents "loss of tractive forces" due to inertia of the engine & drive train components. This term in bracket represents the equivalent inertia of component is amplified by the square of Gear ratio.

we know the eqn,

$$M a_x = \frac{W}{g} a_x = F_x - R_x - D_a - R_{hx} - W \sin\theta$$

By considering engine torque & rotational inertia.

$$\boxed{(M+M_r)a_x = \left(\frac{W+W_r}{g}\right)a_x = \frac{T_e N_{tf} \eta_{tf}}{r} - R_x - D_A - R_{hx} - W\sin\theta}$$

(31)

where,

M_r = equivalent mass of rotating components

M = mass of vehicle.

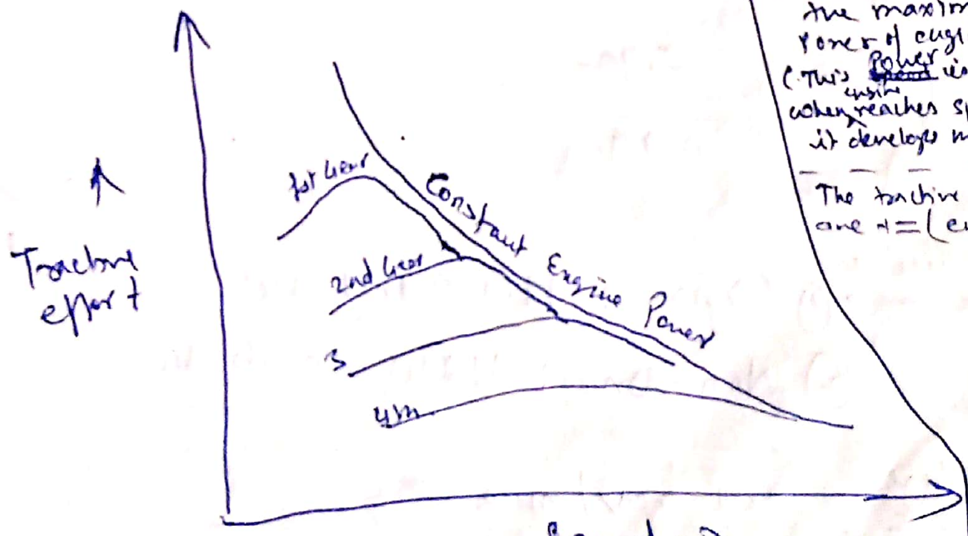
$(M+M_r)$ = effective mass

$\frac{(M+M_r)}{M}$ = mass factor

The mass factor can be ~~determined~~ determined by,

$$\boxed{\text{mass factor} = 1 + 0.04 + 0.0025\, N_{tf}^2}$$

The traction force generated by engine/power train i.e $\left(\frac{T_e N_{tf} \eta_{tf}}{r}\right)$ is the effort available to overcome load & accelerate the vehicle.



Constant power line provided by engine is the maximum power of engine } First term. $\frac{T_e N_{tf} \eta_{tf}}{r}$

(This ~~speed~~ power is achieved when engine reaches speed which it develops max^m power)

The tractive gear lines are = (engine max torque) × (gear ratio)

For maximum acceleration performance the optimum shift point between the gears is the point where line cross. The area between the lines for different gears & the constt power curve is indicative of deficiencies of the transmission in providing max^m power action.

Ans Information about engine & drive train components of passenger car

Engine Inertia 0.8 in-lb-sec

RPM/Torque					
800	120	2400	175	4000	200
1200	132	2800	181	4400	201
1600	145	3200	190	4800	198
2000	160	3600	198	5200	180

Transmission Data -

Gear	1	2	3	4	5	
Inertia.s	1.3	0.9	0.7	0.5	0.3	in-lb-sec^2
Ratios	4.28	2.79	1.83	1.36	1.00	
η	0.966	0.967	0.972	0.973	0.970	

Final drive

Inertia 1.2 in-lb.-sec^2

Ratio 2.92

η 0.99

Wheel Inertia
(1) Drive : 11.00 in-lb-sec^2
(2) Non-Drive : 11.00 in-lb-sec^2

Wheel size 801 rev/mile

Radius = 12.59 inch.

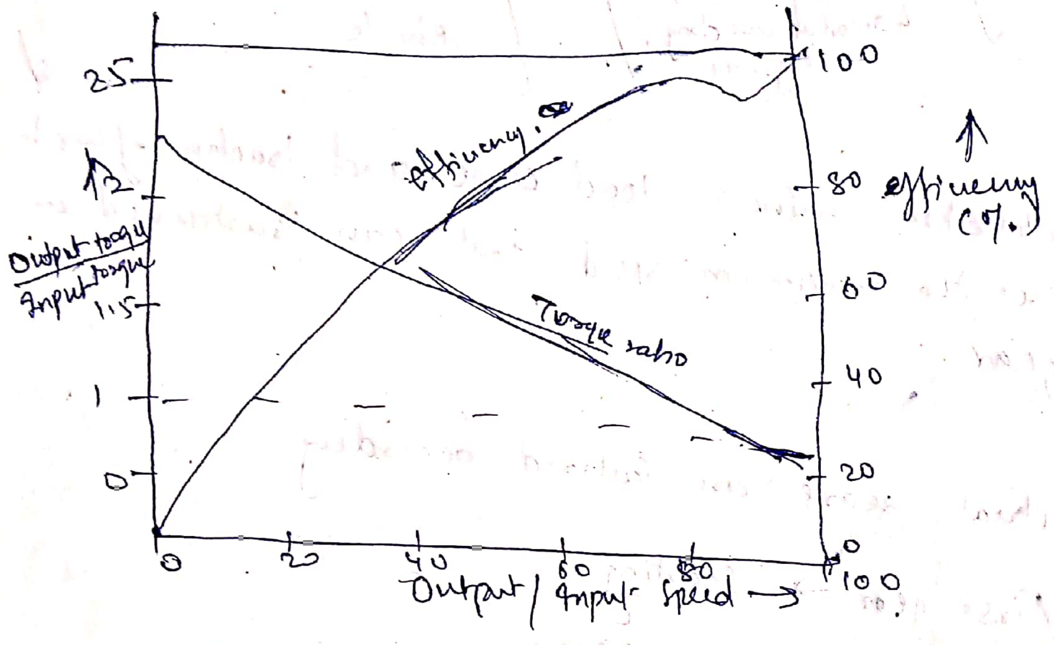
Quest Calculate effective inertia of the drive train component in 1st gear.

* Automatic Transmission :-

32

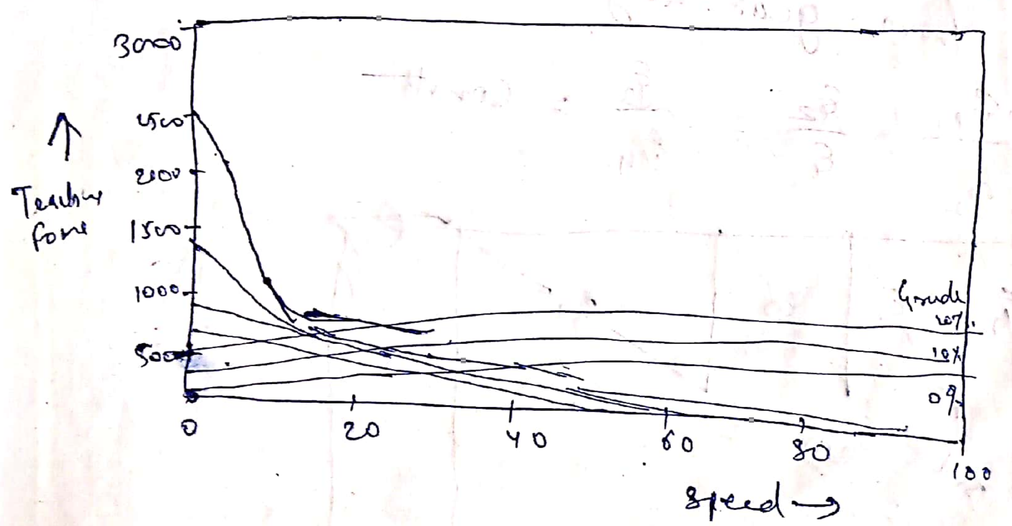
- Automatic transmission provides ideal performance because of torque converter on input.

- Torque converter increase torque at the expense of speed.



① at zero o/p speed, torque at o/p will [2 (engine torque)]
This is condition when transmission is stalled.

② As speed builds up torque ratio drops to unit.



Because of torque amplification more favourable tractive effort-speed performance is achieved.

- As there may be slip condition in torque converter the torque may extend to zero.

~~The~~ at a given speed the difference between the traction effort curve and appropriate load curve is tractive ~~effort~~ as force available to accelerate

[Tractive effort curve] − [Load curve due to rolling resistance, aerodynamic drag, road grade] = [Tractive force available to accelerate the vehicle]

- The intersection between load curve and tractive effort curve is the maximum speed that can sustained in that gear.

The actual gears are tailored according

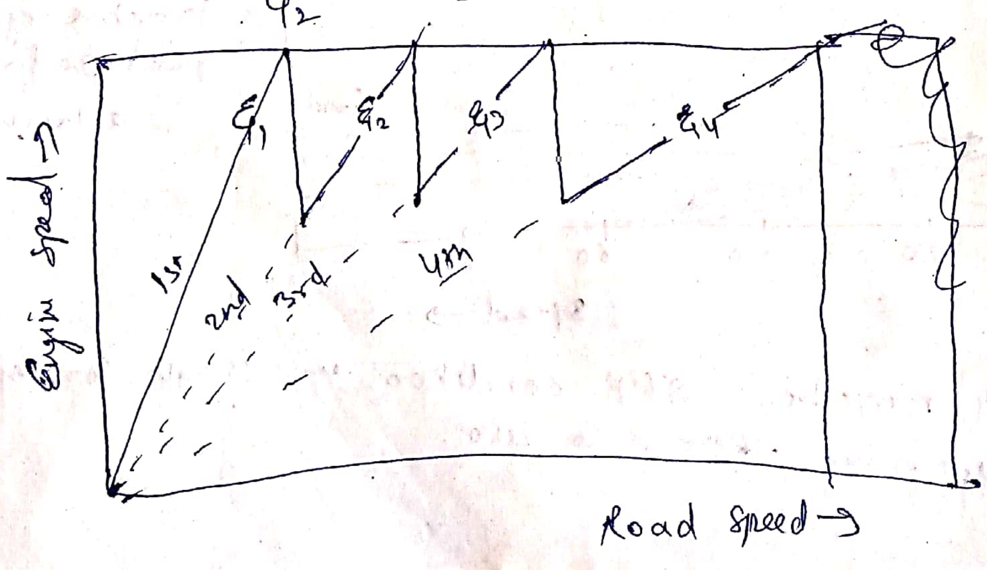
First gear → starting.

Second or third gear → passing.

High gear → fuel economy on roads.

- The best gear ratio usually falls in geometric progression in which ratio change by a constant percent from gear to gear

$$\frac{\xi_{g1}}{\xi_{g2}} = \frac{\xi_{g2}}{\xi_{g3}} = \frac{\xi_{g3}}{\xi_{g4}} = \text{Constt.}$$



$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r} - \left\{ (I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w \right\} \frac{a_x}{r^2}$$

effective inertia,

$$I_{eff} = (I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w.$$

$$= (0.8 + 1.3)(4.28 \times 2.92)^2 + (1.2 \times 2.92^2) + (2 \times 11)$$

$$\boxed{I_{eff} = 360.2 \text{ in-lb-sec}^2}$$

* Traction Limited Acceleration :-

Let us assume that there is adequate power from the engine. The acceleration is limited by

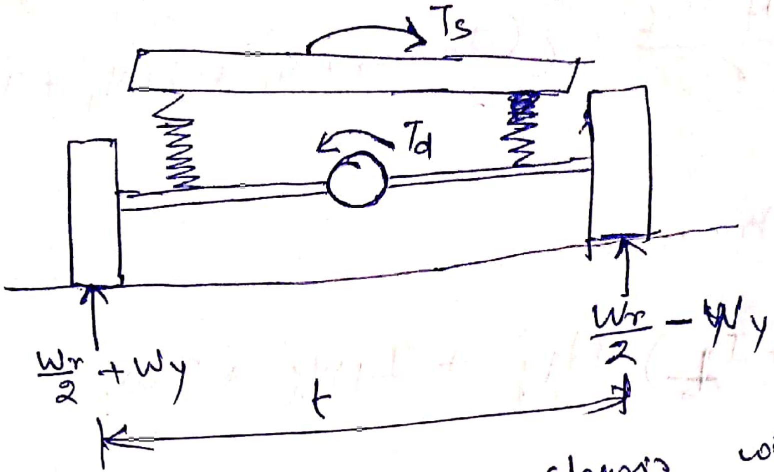
$F_x = \mu W.$ { where μ = Peak coeff. of friction, W = wt. on drive wheels.

But the wt. on drive wheel depend (W) :-

- Static + dynamic load due to accel^n (This is already calculated)
- transverse shift of load due to drive torque.

* Transverse shift of load due to drive torque

- Transverse shift occurs in both solid axles front & rear
- driveshaft ~~imposes~~ in differential gives T_d torque on axle.



— due to T_d torque, the chassis will roll compressing & extending springs on opposite sides of vehicle such that a torque due to suspension roll stiffness T_s is produced.

— ~~different~~ difference between T_d & T_s will be absorbed by road wheel.

— If the axle is non-locking type, then the torque delivered to both wheels will be limited by the traction limit on the most lightly load wheel.

Taking moment about (c) ~~Centre~~ centre for eqlb^m condition,

$$\sum T_o = 0 = \left\{\left(\frac{W_r}{2} + W_y\right) - \left(\frac{W_r}{2} - W_y\right)\right\}\frac{t}{2} + T_s - T_d$$

↻ +ve
↺ −ve

$$\left\{\cancel{\frac{W_r}{2}} + W_y - \cancel{\frac{W_r}{2}} + W_y\right\}\frac{t}{2} + T_s - T_d = 0$$

$$\cancel{2}W_y \cdot \frac{t}{\cancel{2}} + T_s - T_d = 0$$

$$\boxed{W_y = \frac{T_d - T_s}{t}}$$

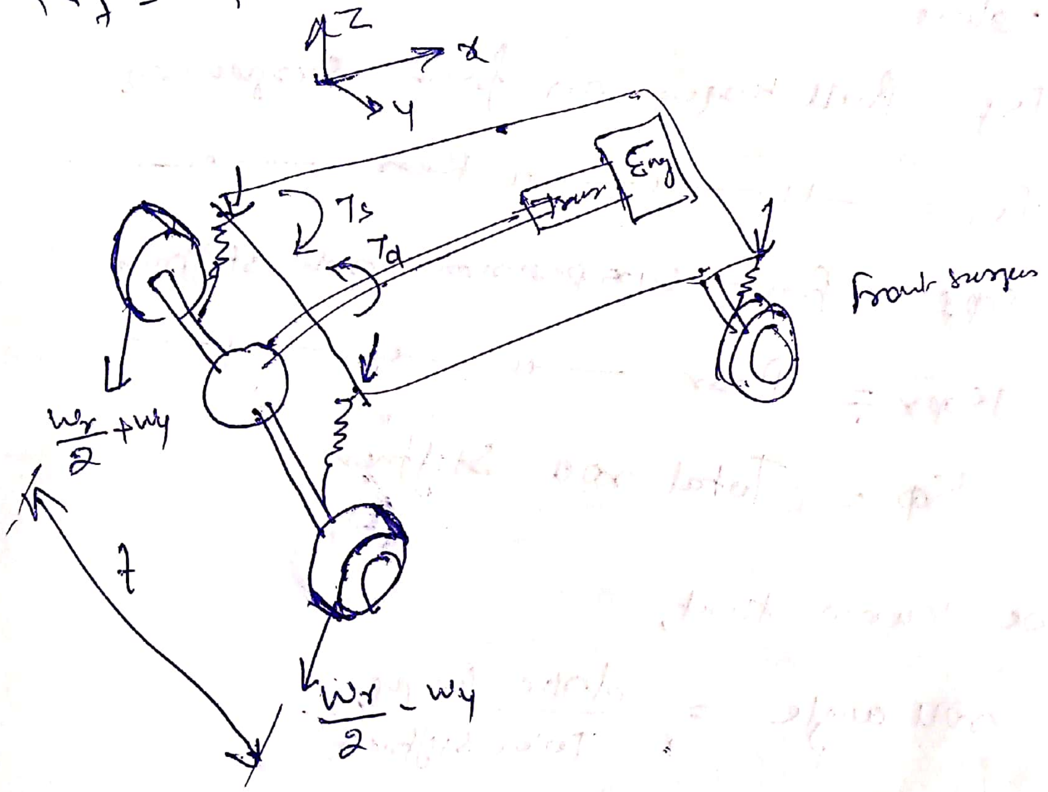
The differential torque can also be written as

$$\boxed{T_d = \frac{F_x \cdot r}{N_f}}$$

F_x = Total drive ~~force of~~ force from both wheels

r = Tire radius

N_f = final drive gear ratio.



- The drive torque reaction at engine/~~to~~ transmission is transferred to frame and distributed between front & rear suspension.

It is assumed that ~~roll torque~~

$$\boxed{\text{roll torque} = \text{roll angle of chassis}}$$

By hooks law,

then, $T_{sf} = K_{\phi f} \cdot \phi$

$$T_{sf} = (K\phi_f) \cdot \phi$$

$$T_{sr} = (K\phi_r)\,\phi$$

and we know that

Total stiffness, $K\phi = (K\phi_f) + (K\phi_r)$

where,

T_{sf} = Roll torque on front suspension.

T_{sr} = — " — " — " Rear — " —

$K\phi_f$ = front suspension roll stiffness

$K\phi_r$ = Rear — " — " — " —

$K\phi$ = Total roll stiffness.

we know that,

$$\text{roll angle} = \frac{\text{drive Torque}}{\text{Total stiffness.}}$$

$$\phi = \frac{T_d}{K\phi}$$

$$\boxed{\phi = \frac{T_d}{(K\phi_f + K\phi_r)}}$$

therefore.

$$T_{sr} = (K\phi_r) \cdot \phi$$

$$\boxed{T_{sr} = (K\phi_r) \cdot \left(\frac{T_d}{K\phi_f + K\phi_r}\right)}$$

and we know

$$W_y = \frac{T_d - T_s}{t} \quad \& \quad T_d = \frac{F_x \cdot r}{N_f}$$

~~So,~~

$$\cancel{W_y = \frac{F_x \cdot r}{(N_f)\, t} - T_{sr}}$$

$$\cancel{W_y = \frac{F_x \cdot r}{(N_f)\, t} - \frac{(K\phi r)\, T_d}{(K\phi f + K\phi r)}}$$

$$\cancel{W_y = F_x \cdot r}$$

$$\therefore \quad W_y = \frac{T_d - T_s}{t}$$

$$W_y = \frac{T_d}{t} - \frac{(K\phi r)\, T_d}{(K\phi f + K\phi r)\, t}$$

$$W_y = \frac{T_d}{t}\left[1 - \frac{(K\phi r)}{(K\phi f) + (K\phi r)}\right]$$

$$W_y = \frac{F_x \cdot r}{N_f \cdot t}\left[1 - \frac{(K\phi r)}{K\phi f + K\phi r}\right]$$

$$W_y = \frac{F_x \cdot r}{N_f \cdot t}\left[\frac{K\phi f + \cancel{K\phi r} - \cancel{K\phi r}}{K\phi f + K\phi r}\right]$$

$$\boxed{W_y = \left(\frac{F_x \cdot r}{N_f \cdot t}\right)\left(\frac{K_{\phi f}}{K_\phi}\right)}$$

Now,

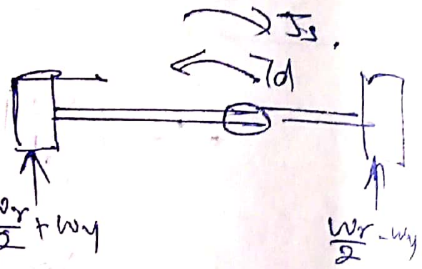
we know that by dynamic axle loading condition that

$$W_r = W\left(\frac{b}{L} + \frac{a_x h}{gL}\right)$$

$$\boxed{W_r = W\left(\frac{b}{L} + \frac{F_x h}{MgL}\right)}$$

then ~~wt~~ weight on right rear wheel

$$W_{rr} = \frac{W_r}{2} - W_y$$



$$W_{rr} = \frac{W}{2}\left[\frac{b}{L} + \frac{F_x h}{mgL}\right] - \left[\left(\frac{F_x \cdot r}{N_f \cdot t}\right)\left(\frac{K_{\phi f}}{K_\phi}\right)\right]$$

$$W_{rr} = \left[\frac{W b}{2L} + \frac{\cancel{W} F_x h}{2\cancel{mg} L}\right] - \left[\left(\frac{F_x \cdot r}{N_f \cdot t}\right)\left(\frac{K_{\phi f}}{K_\phi}\right)\right]$$

$$\boxed{W_{rr} = \frac{Wb}{2L} + \frac{F_x \cdot h}{2L} - \left(\frac{F_x \cdot r}{N_f \cdot t} \cdot \frac{K_{\phi f}}{K_\phi}\right)}$$

and

~~Fx~~ $F_x = 2\mu W_{rr}$

$F_x = 2\mu W_{rr}$

$$\boxed{F_x = 2\mu\left[\frac{Wb}{2L} + \frac{F_x h}{2L} - \frac{F_x r}{N_{f\tau}} \cdot \frac{K_{\phi f}}{K_\phi}\right]}$$

Traction limits

For solid rear axle with non-locking differential,

$$\boxed{(F_x)_{max} = \frac{\left(\mu \frac{Wb}{L}\right)}{1 - \frac{h}{L}\mu + \left(\frac{2\mu r}{N_{f\tau}} \frac{K_{\phi f}}{K_\phi}\right)}}$$

But for solid rear axle with locking different,

$$\boxed{(F_x)_{max} = \frac{\left(\mu \frac{Wb}{L}\right)}{1 - \frac{h}{L}\mu}}$$

In this case additional tractive force is obtained from other wheels, upto its traction limit
– It will also applicable for independent suspension because the T_d is placed by chassis mounted differential

Similarly for rear wheels,

For non-locking differential,

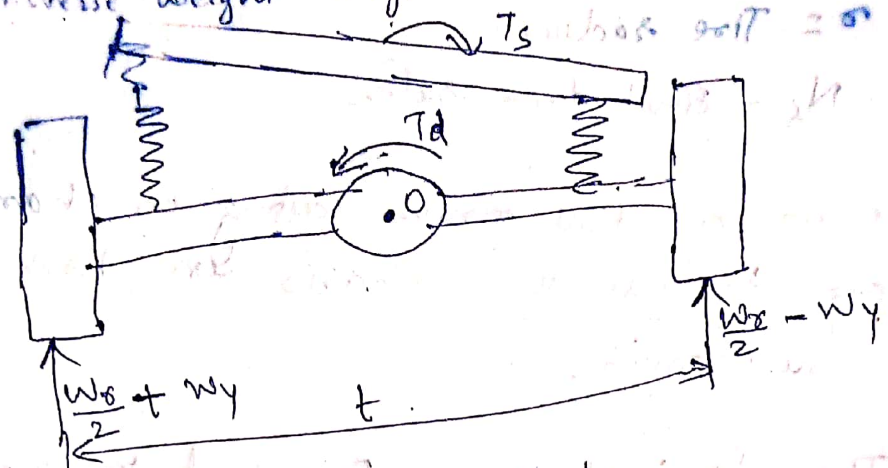
$$\boxed{(F_x)_{max} = \frac{\mu \frac{Wc}{L}}{1 + \frac{h\mu}{L} + \frac{2\mu r}{N_{f\tau}} \frac{K_{\phi r}}{K_\phi}}}$$

For locking

$$\boxed{(F_x)_{max} = \frac{\mu Wc/L}{1 + \frac{h}{L}\mu}}$$

Transverse weight shift due to drive torque

Transverse weight shift occurs on all drive axles.



T_d = Torque due to drive shaft

T_s = Torque due to roll of chassis.

W_r = Total wt. on rear axle

W_y = The resultant force generated on wheels due to difference in drive torque and chassis torque.

t = wheel track.

Assuming the axle is non-locking type (same torque on each wheel & allowing to rotate with different speed)

Using Newton's 2nd law, Taking moment at O for equlb^m,

$$\left(\frac{W_r}{2} + W_y\right)\frac{t}{2} - \left(\frac{W_r}{2} - W_y\right)\frac{t}{2} = T_d + T_s$$

$$\Sigma T_O = 0 = \left(\cancel{\frac{W_r}{2} \cdot \frac{t}{2}}\right) + \left(W_y \cdot \frac{t}{2}\right) - \left(\cancel{\frac{W_r}{2} \cdot \frac{t}{2}}\right) + \left(\frac{W_y \cdot t}{2}\right) - T_d + T_s = 0$$

$$\cancel{2}\left(W_y \cdot \frac{t}{\cancel{2}}\right) - T_d + T_s = 0$$

$$\boxed{W_y = \frac{(T_d - T_s)}{t}} \quad \text{——— (1)}$$

But we know,

$$\boxed{T_d = \frac{F_x \cdot r}{N_f}} \quad \text{——— (2)}$$

F_{td} = Total drive tractive force

r = Tire radius

N_f = final drive ratio.

There is a Roll torque acting on chassis attempting to roll the chassis on both front & rear suspension.

The drive torque is transferred to the suspension system of chassis in front & rear.

By hooke's law of spring (i.e $F_x = kx$)

Similarly

$$T_{sf} = K_{\phi f} \cdot \phi$$

$$T_{sr} = K_{\phi r} \cdot \phi$$

$$K_\phi = K_{\phi f} + K_{\phi r}$$

where,

T_{sf} = Roll Torque on front suspension

T_{sr} = —"—"—"— rear —"—

ϕ = Roll angle

K_ϕ = ~~torque~~ is total roll stiffness

$K_{\phi f}$ = Front suspension roll stiffness

$K_{\phi r}$ = rear —"— —"— —"—

we also know,

roll angle, $\boxed{\phi = \dfrac{T_d}{K_\phi}}$ —③

$$\boxed{\varphi = \frac{T_d}{(K_{\varphi f} + K_{\varphi r})}}$$

~~putting above value of roll angle~~

$$W_y = \left(\frac{T_d}{t} - \frac{T_s}{t}\right)$$

$$= \left(\frac{F_x \cdot r}{N_f t} - \frac{T_s}{t}\right)$$

$\because T_{sr} = K_{\varphi r} \cdot \varphi$

$$\boxed{T_{sr} = K_{\varphi r}\left[\frac{T_d}{(K_{\varphi f} + K_{\varphi r})}\right]} \longrightarrow (4)$$

Substituting equ (4) in (1)

$$\therefore W_y = \left(\frac{T_d - T_s}{t}\right)$$

$$W_y = \frac{T_d}{t} - \frac{T_s}{t}$$

$$W_y = \left[\frac{T_d}{t} - \frac{K_{\varphi r}}{t}\left[\frac{T_d}{(K_{\varphi f} + K_{\varphi r})}\right]\right]$$

Substituting equ (2)

$$W_y = \frac{T_d}{t}\left[1 - \frac{K_{\varphi r}}{(K_{\varphi f} + K_{\varphi r})}\right]$$

$$W_y = \frac{F_x \cdot r}{N_f \cdot t}\left[1 - \frac{K_{\varphi r}}{(K_{\varphi f} + K_{\varphi r})}\right]$$

$$W_y = \frac{F_x \cdot r}{N_f \cdot t}\left[\frac{K_{\varphi f} + \cancel{K_{\varphi r}} - \cancel{K_{\varphi r}}}{K_{\varphi f} + K_{\varphi r}}\right]$$

$$W_y = \frac{F_x \cdot r}{N_f \cdot t}\left[\frac{K_{\varphi f}}{K_{\varphi f} + K_{\varphi r}}\right]$$

$$W_y = \frac{F_x \cdot r \cdot K_{\varphi f}}{N_f \cdot t \cdot K_\varphi} \cdot \varphi$$

This equation gives the magnitude of lateral load transfer as a function of tractive force & a numbers of parameters such N_f, t, r, φ etc.

So, the net load on the rear axle will be (static + dynamic load)

So,

$$W_r = W\left[\frac{b}{L} + \frac{a_x \cdot h}{g \cdot L}\right]$$ { neglecting R_x, D_x etc.

$$\boxed{W_r = W\left[\frac{b}{L} + \frac{F_x \cdot h}{M \cdot g \cdot L}\right]}$$

So the weight on the right wheel will be as we are considering minimum tractive force

$$W_{rr} = \frac{W_r}{2} - W_y \quad \text{—— from fig.}$$

$$= \left(\frac{Wb}{2L} + \frac{\cancel{W} F_x \cdot h}{2MgL}\right) - \frac{F_x \cdot r \cdot K_{\varphi f}}{N_f \cdot t \cdot K_\varphi}$$

$$\boxed{W_{rr} = \frac{W \cdot b}{2L} + \frac{F_x \cdot h}{2L} - \frac{F_x \cdot r \cdot K_{\varphi f}}{N_f \cdot t \cdot \varphi}}$$

So the min. tractive force on Right ~~rear~~ wheel will be,

$$F_x = 2\mu W_{rr}$$

$$\boxed{F_x = 2\mu\left[\frac{Wb}{2L} + \frac{F_x \cdot h}{2L} - \frac{F_x \cdot r\, K_{\varphi f}}{N_f \cdot t \cdot \varphi}\right]}$$

Traction Limits

∴ Solving F_x for its maximum traction force that can be developed by a solid rear axle with a non-locking differential, (39)

$$F_{xmax} = 2\mu\left[\frac{Wb}{2L} + \frac{F_{xmax}\cdot h}{2L} - \frac{F_{xmax}\cdot r\cdot K_{\phi f}}{N_f\cdot t\cdot K_{\phi}}\right]$$

$$F_{xmax} = \frac{2\mu Wb}{2L} + \frac{2\mu F_{xmax}\cdot h}{2L} - \frac{2\mu F_{xmax}\cdot r\cdot K_{\phi f}}{N_f\cdot t\cdot K_{\phi}}$$

$$F_{xmax} - \frac{2\mu F_{xmax}\cdot h}{2L} + \frac{2\mu F_{xmax}\cdot r\cdot K_{\phi f}}{N_f\cdot t\cdot K_{\phi}} = \frac{2\mu Wb}{2L}$$

$$F_{xmax}\left[1 - \frac{\mu\cdot h}{L} + \frac{2\mu r\cdot K_{\phi f}}{N_f\cdot t\cdot K_{\phi}}\right] = \frac{\mu W\cdot b}{L}$$

$$\boxed{F_{xmax} = \frac{\left(\frac{\mu W b}{L}\right)}{1 - \frac{\mu\cdot h}{L} + \frac{2\mu r\cdot K_{\phi f}}{N_f\cdot t\cdot K_{\phi}}}}$$

For a solid rear axle with locking differential, additional traction force is obtained from the other wheel, so that the third term in denominator cancels out.

The similar case ~~also~~ independent rear suspension because the driveline torque reaction is picked up by chassis-mounted differential.

Hence expression for both cases is,

$$\boxed{F_{xmax} = \frac{\left(\frac{\mu Wb}{L}\right)}{1 - \left(\frac{\mu h}{L}\right)}}$$

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~~solid~~ solid ~~drive~~ front drive axle

- locking differential,

$$\cancel{F_x max = 2\mu \left[\frac{WC}{2L} - \frac{F_x . h}{2L} - F_x . r . K\phi \right]}$$

The equtn can be written as,

$$\boxed{F_x max = \frac{\frac{\mu W C}{L}}{1 + \frac{\mu h}{L} + \frac{2\mu r\, K\phi r}{N_f . t . K\phi}}}$$

and the equtn for solid front drive axle with locking differential or the independent front drive axle as most cars today are front wheel drive.

$$\boxed{F_x max = \frac{\frac{\mu W C}{L}}{1 + \frac{\mu h}{L}}} \quad - 1$$

Page No.
Date

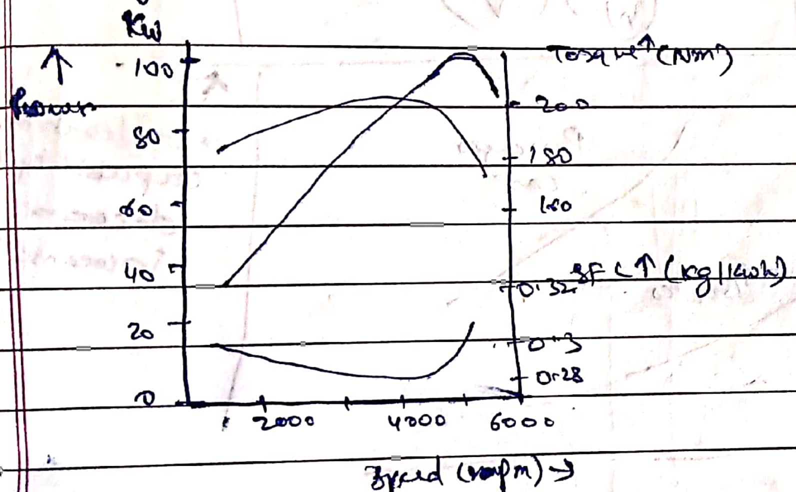
Unit-5

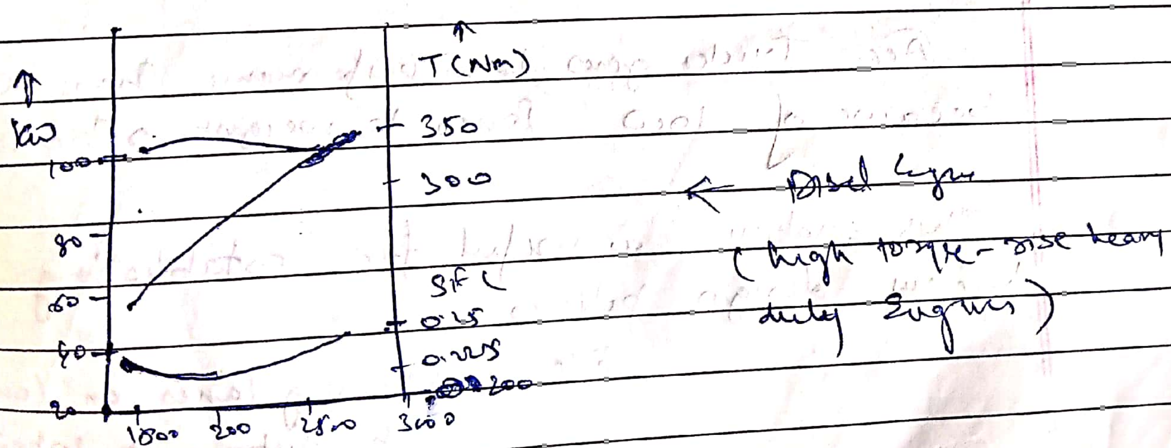
Maximum performance in longitudinal accel^n in vehicle determined by two limits
→ (1) Engine Power (at high speed it is limiting factor)
→ (2) Traction limit on drive wheels (at low speed it is limiting factor)

(1) Engine Power limited Accleration

It involves examination of engine and interaction of engine with power train.

(a) Engine:





$P = T \cdot \omega$

The ratio of Engine Power to vehicle weight is the first order determinant of acceleration Performance.

at low speed.,

By Newton 2nd law,

Tractive force,

$$F_x = m \cdot a_x$$

$$a_x = \frac{F_x}{m} \quad \text{--- (1)}$$

$$\therefore \text{Power} = F_x \cdot V$$

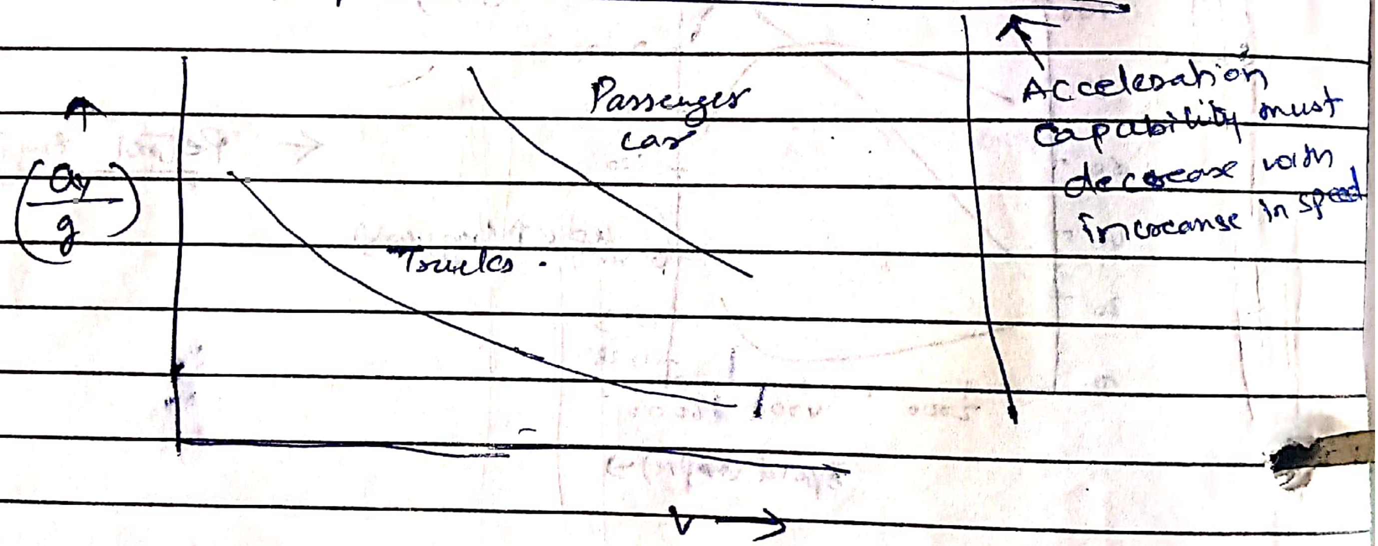
$$\text{and } W = m \cdot g.$$

So eqn (1) becomes

$$\boxed{a_x = \left(\frac{P}{V}\right)\left(\frac{g}{W}\right) \text{ m/s}^2}$$

or

$$\boxed{a_x = \left(\frac{P}{W}\right)\left(\frac{g}{V}\right) \text{ m/s}^2}$$



Acc. Trucks gives low performance than Car because of low Power to weight ratio.

This analysis is useful for establishing highway design policies

like:- Climbing lanes on long upgrades
- Sight distance at Intersection
- accl^n areas on entrance ramp.

Transient operations

~~maximum performance in lo~~

Performance of motor vehicle in longitudinal acceleration is given by two limits

- Engine power (at high speed considered)
- traction limits (drive wheel) (at low speed considered)

(I) Engine power limited acceleration

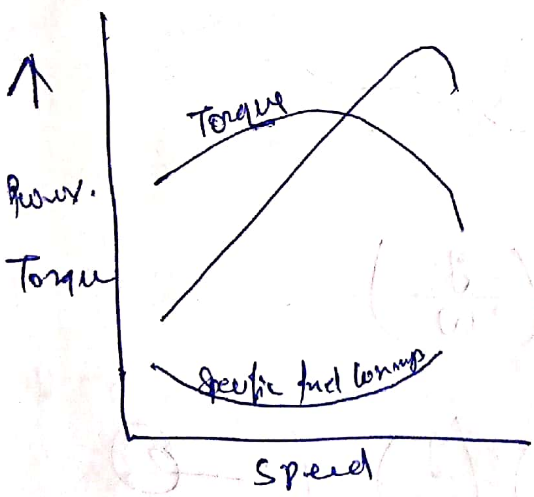
— It involves examination of engine characteristics and their interaction with power train

(A) Engines

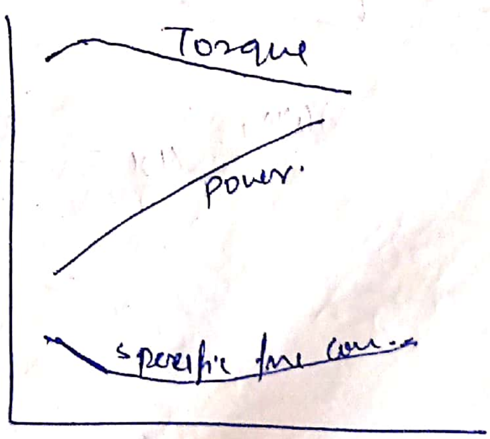
characterized by

(1) Torque curves
(2) Power curves

} as a function of speed.



Petrol Engine.



Diesel Engine

ratio of Engine power to vehicle wt is the first-order determinant of acceleration performance.

— At low to moderate speed,

acceleration can be obtained by neglecting all resistance forces acting on vehicle.

By, Newton's 2nd law,

$$\boxed{F_x = m a_x} \quad \text{—— (1)}$$

where

m = mass of vehicle $= \frac{W}{g}$

a_x = acceleration in forward direction.

F_x = Tractive force at drive wheels

$\therefore$ $\boxed{\text{drive power} = \text{Tractive force} \times \text{Forward speed.}}$

Then,

$$a_x = \frac{F_x}{m}$$

$$= \left(\frac{HP}{V}\right) \cdot \left(\frac{g}{W}\right)$$

$$\boxed{a_x = \left(\frac{P}{V}\right)\left(\frac{g}{W}\right)} \quad \text{—— (2)}$$

where $g = 9.81 \, m/s^2$

V = ~~veloc~~ forward velocity

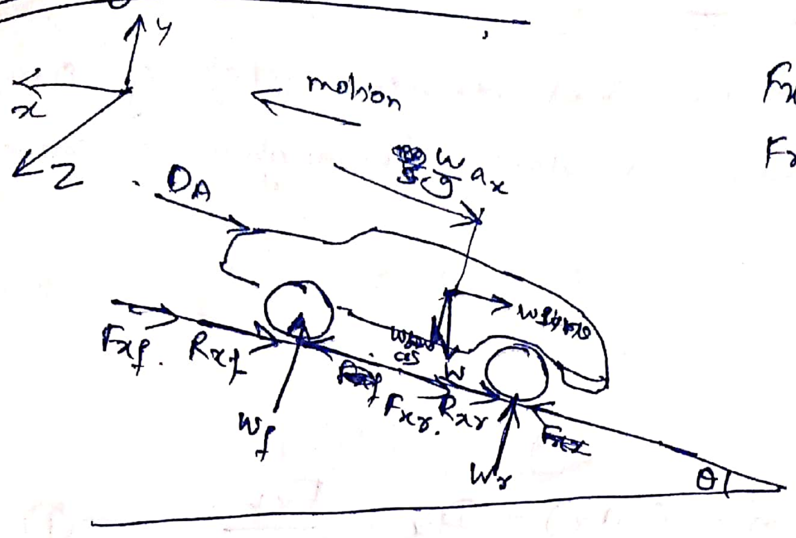
P = Engine Power in ~~kw~~ W.

W = wt in Kg.

* Braking Performance

$F_{xf} =$ } Braking force opposite to motion will act
$F_{xr} =$ }

(70)



~~$Ma_x = \frac{W}{g}a_x$~~

here during braking ~~acc~~ deceleration or retardation will take place so, that acceleration will be $(-a_x)$

So, resolving along x-axis for equilibrium condition

$$\Sigma f_x = 0 = -\frac{W}{g}(-a_x) - D_A - W\sin\theta - F_{xf} - F_{xr}$$

$$0 = \frac{W}{g}a_x - D_A - W\sin\theta - F_{xf} - F_{xr}$$

$$\boxed{-\frac{W}{g}a_x = -F_{xf} - F_{xr} - D_A - W\sin\theta}$$

This is the general equation for braking Performance

* The F_{xf} & F_{xr} will consist of Braking torque, Rolling Resistance, bearing friction & driveline drag.

In order to do the proper analysis we should know all above forces.

Constant Deceleration

We can assume that forces acting on the vehicle will be constant throughout brake application.

we know;

$$\text{deceleration} = (-a_x) = D_x = \frac{F_{xt}}{M} \quad \text{—— (1)}$$

$$\text{but} \quad D_x = -\frac{dv}{dt} \quad \text{—— (2)}$$

where F_{xt} = Total longitudinal deceleration force acting on vehicle.

V = forward velocity.

Equating eqution (1) & (2) & integrating velocity w.r.t time.

$$\frac{F_{xt}}{M} = -\frac{dv}{dt}$$

→ (1) relation between velocity & time

$$dv = -\frac{F_{xt}}{M} \cdot dt$$

$$\int_{V_0}^{V_f} dv = -\frac{F_{xt}}{M} \int_0^{t_s} dt$$

$$[V]_{V_0}^{V_f} = -\frac{F_{xt}}{M} [t]_0^{t_s}$$

$$\boxed{[V_f - V_0] = -\frac{F_{xt}}{M} [t_s]}$$

where V_f = final velocity & t_s = time for velocity change
V_0 = Initial velocity

Relation between velocity & distance

Now again taking equn

$$dv = -\frac{F_{xt}}{M}\,dt$$

but we know that $v = \frac{dx}{dt}$

So, $dt = \frac{dx}{v}$

Substituting in above equn

$$dv = -\frac{F_{xt}}{M}\cdot\frac{dx}{v}$$

$$v\cdot dv = -\frac{F_{xt}}{M}\cdot dx$$

Integrating with the limits

$$\int_{v_0}^{v_f} v\cdot dv = -\frac{F_{xt}}{M}\int_0^x dx$$

$$\left[\frac{v^2}{2}\right]_{v_0}^{v_f} = -\frac{F_{xt}}{M}\left[x\right]_0^x$$

$$\boxed{\frac{1}{2}\left[v_f^2 - v_0^2\right] = -\frac{F_{xt}}{M}\left[x\right]}$$

where v_f = final velocity

v_0 = initial —u—

x = Distance travelled during deceleration.

In the case when deceleration results in stop

then $v_f = 0$.

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Constant Deceleration

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$$\frac{F_{xt}}{M} = -\frac{dv}{dt}$$

→ (1) relation between velocity & time

$$dv = -\frac{F_{xt}}{M} \cdot dt$$

$$\int_{V_0}^{V_f} dv = -\frac{F_{xt}}{M} \int_0^{t_s} dt$$

$$[v]_{V_0}^{V_f} = -\frac{F_{xt}}{M} [t]_0^{t_s}$$

$$\boxed{[V_f - V_0] = -\frac{F_{xt}}{M} [t_s]}$$

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Relation between velocity & distance

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So, $dt = \frac{dx}{v}$

Substituting in above equn

$$dv = -\frac{F_{xt}}{M} \cdot \frac{dx}{v}$$

$$v \cdot dv = -\frac{F_{xt}}{M} \cdot dx$$

Integrating with the limits

$$\int_{v_0}^{v_f} v \cdot dv = -\frac{F_{xt}}{M} \int_0^x dx$$

$$\left[\frac{v^2}{2}\right]_{v_0}^{v_f} = -\frac{F_{xt}}{M} \left[x\right]_0^x$$

$$\boxed{\frac{1}{2}\left[v_f^2 - v_0^2\right] = -\frac{F_{xt}}{M}\left[x\right]}$$

where v_f = final velocity

v_0 = initial — " —

x = Distance travelled during deceleration.

In the case when decleration results in stop

then $v_f = 0$.

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Stopping distance,

$$\frac{V_0^2 - 0^2}{2} = \frac{F_{xt}}{m} \cdot x$$

$$\frac{V_0^2}{2} = \frac{F_{xt}}{m} \cdot x$$

$$x = \frac{V_0^2}{2 \cdot \left(\frac{F_{xt}}{m}\right)}$$

$$\boxed{x = \frac{V_0^2}{2 D_x}}$$ — stopping distance.

& time to stop is,

we know

$$(V_f - V_0) = -\frac{F_{xt}}{m}(t_s)$$

$$(0 - V_0) = -\frac{F_{xt}}{m}(t_s)$$

$$\cancel{-} V_0 = \cancel{-} \frac{F_{xt}}{m}(t_s)$$

$$V_0 = D_x (t)$$

$$\boxed{t = \frac{V_0}{D_x}}$$ — time to stop the vehicle.

Thus, (time to stop) $\propto$ (velocity)

whereas, (distance to stop) $\propto$ (square of velocity)

* Deceleration with Wind Resistance

The aerodynamic drag force will be assisting the braking force. So that the total braking will be,

$$F_x = (\text{braking force}) + (\text{Aerodynamic drag forces})$$

$$\boxed{F_x = (F_b) + Cv^2}$$

where,

F_b = Total brake force of front & rear wheels

C = Aerodynamic drag factor.

But we know that,

$$(-a_x) = D_x = \frac{F_{xt}}{M}$$

and $a_x = \frac{-dv}{dt}$

So combing

$$\frac{F_{xt}}{M} = -\frac{dv}{dt}$$

and $v = \frac{dx}{dt}$

$$dt = \frac{dx}{v}$$

So,

$$\frac{F_{xt}}{M} = -\frac{v \cdot dv}{dx}$$

$$dx = -M \int \frac{v \cdot dv}{F_{xt}}$$

$$dx = -M \int \frac{v \cdot dv}{F_b + Cv^2}$$

$$\int_0^x dx = -\int_{V_0}^{0} M \left(\frac{V\,dv}{F_b + CV^2}\right)$$

$$x = -\frac{M}{2C}\left\{\log\left[\frac{F_b + CV_0^2}{F_b}\right]\right\}_{V_0}^{0}$$

$$\boxed{x = \frac{M}{2C}\log\left(\frac{F_b + CV_0^2}{F_b}\right)}$$

let

$F_b + CV^2 = x$

$0 + 2CV\cdot dv = dx$

$V\cdot dv = \frac{dx}{2C}$

$-\int_{V_0}^{0} \frac{dx}{2C}\cdot\frac{1}{x}$

$-\frac{1}{2C}\int_{V_0}^{0} \frac{dx}{x}$

$-\frac{1}{2C}\{\log x\}$

$-\frac{1}{2C}(\log F_b + CV^2)$

$= -\frac{1}{2C}\{\log F_b - \log F_b \ldots\}$

$= \frac{1}{2C}\log\left(\frac{F_b + CV_0^2}{F_b}\right)$

stopping distance when Aerodynamic drag Consider.

* Energy / Power

When maximum effort is required to stop the vehicle a substantial amount of power ~~of power~~ & energy of braking is required.

We know that Energy absorbed by a body in stopping other body in motion is given by.

$$\text{Energy} = \frac{1}{2}M(V_0^2 - V_f^2)$$

Similarly the power absorbed during braking to stop the vehicle is given by,

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$= \frac{1}{2t_s}\{M(V_0^2 - V_f^2)\}$$

$$P = \frac{1}{2t_s}\{MV_0^2\}$$

$$\boxed{P = \frac{M V_0^2}{2 t_s}}$$

For e.g.

A Car,

$m = 3000\ lb$

$V_0 = 80\ mph$

Energy required = 650,000 ft-lb

$t_s = 8$ sec

$V_f = 10$ mph/sec.

average power absorbed by brake will be 145 Hp.

Braking Forces

The Forces on vehicle producing braking deceleration may come from ~~no~~ number of sources

- Primary source — Brake
- Secondary source — Rolling Resistance, Aerodynamic drag, Driveline drag, grade

Secondary Sources

① Rolling Resistance

Rolling resistance is opposite to vehicle motion, hence it aids the brakes

$$R_{xf} + R_{xr} = f_r (W_f + W_r)$$

$= f_r W$

where,

f_r = rolling resistance Coefficient.

W_r = load acting on rear wheel

W_f = — " — " — front — " —

(2) Aerodynamic Drag.

The drag from air resistance depends on dynamic pressure &,

drag $\propto$ (speed)^2

at low speed it is negligible

at high speed, it is in considerable amount of about 0.03 g ~~[illegible]~~

(3) Drive line Drag :-

— Engine, transmission & final drive

Both contribute drag & inertia effect on braking action.

— As discussed in acceleration performance, the inertia of this component adds to effective mass of vehicle. and hence its consideration is taken in braking during on the wheels

— drag → from bearing & gear friction in transmission, differential & engine braking.

— Engine braking is equivalent to dynamometer

— In manual transmission → clutch is engaged hence gear ratio is multiplied to obtained engine braking

Torque Converter → Power is Transfered from engine to driveline but they are ineffective in reverse direction.

hence ... contribute in automatic transmission

- driveline ~~loss~~ drag aids in braking?

It depends on rate of deceleration

(a) if (deceleration of vehicle) > (deceleration of driveline component due to friction between them)

then extra load of braking is required

(b) if (deceleration of vehicle) < (deceleration of drive line components)

then it will aid in braking of vehicle.

(4) Grade

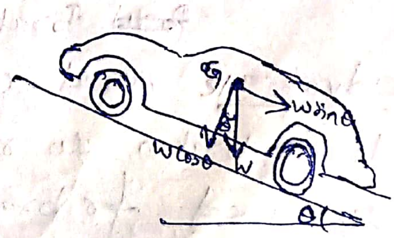
Road grade will contribute directly to the braking effort.

uphill grade → aid in braking (+ve)

downhill grade → more effort in braking (-ve)

grade is defined as rise over the run.

Drag, $R_g = W\sin\theta$



for small angle $\sin\theta \approx \theta$

$$\boxed{R_g = W\theta}$$

So, grade of 4% will be equivalent to deceleration of $\pm 0.04g$.

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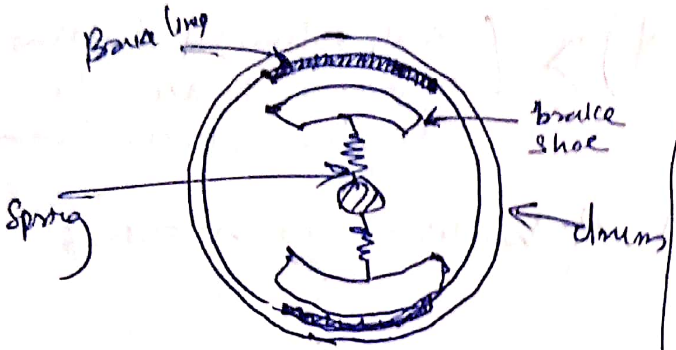
Brakes

Automotive
Brakes

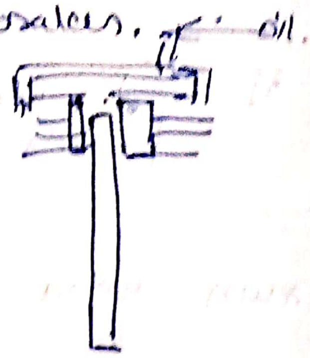
drum
Brakes



① have high brake factor
& easy applied in
parking brake feature

② Not consistent in
torque performance

disc
Brakes.



① low Brake factor

② Due to low brake
factor high actuation
force required

③ Consistent ~~to~~ in
torque performance

④ parking brake
feature is to be
developed.

---

① Brake factor

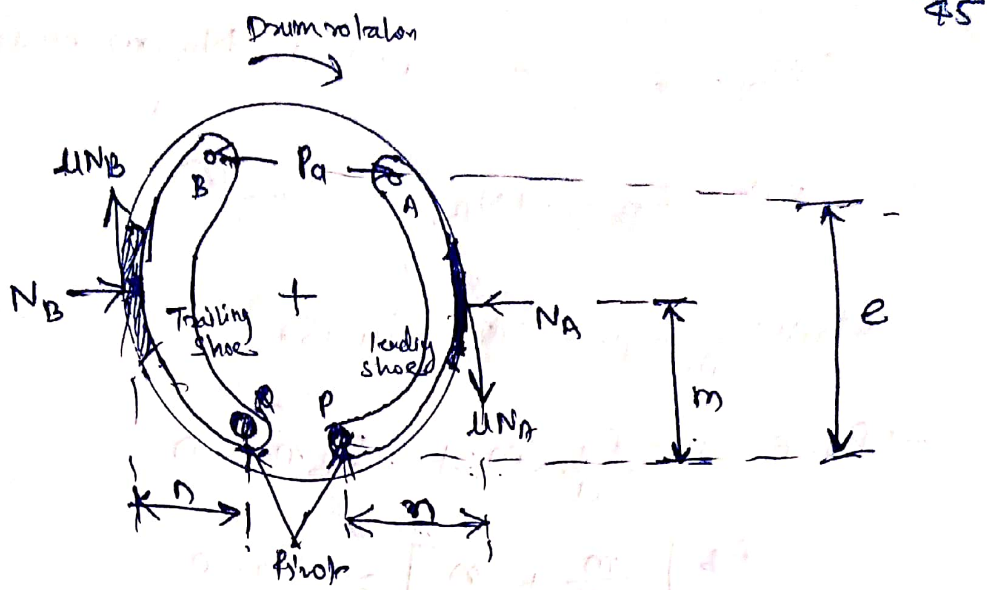
~~Brad~~ Brake factor is a mechanical advantage that can be ~~with~~ utilized in drum brakes to minimize the actuation effort required for braking.

due to actuating force the brake lining pushes on the drum, & hence due to friction brake is applied

$$\text{Friction force} = \mu N_A$$

Pa = Actuation Force to Brake.

calculat? 45



Taking moment @ Pivot Pt. P of shoe A,

$$\Sigma M_P = 0 = e \cdot P_a + (\mu N_a) \cdot n - N_A \cdot m \quad — ①$$

But friction force,

$$F_A = \mu N_A \quad — ②$$

Substituting equtn ② in ①

$$e \cdot P_a + F_A \cdot n - N_A \cdot m = 0$$

$$e \cdot P_a + F_A \cdot n - \frac{F_A}{\mu} \cdot m = 0$$

$$e \cdot P_a + F_A \left[n - \frac{m}{\mu} \right] = 0$$

$$e \cdot P_a + F_A \left[\frac{n\mu - m}{\mu} \right] = 0$$

$$F_A \left[\frac{n\mu - m}{\mu} \right] = -e \cdot P_a$$

$$\frac{F_A}{P_a} = \frac{-e\mu}{n\mu - m}$$

$$\boxed{\frac{F_A}{P_a} = \frac{\mu e}{m - n\mu}}$$

Similarly taking moment @ q. for shoe B.

$$\Sigma M_q = 0 = -P_a \cdot e + N_B \cdot m + \mu N_B n \quad \text{--- (3)}$$

But $F_B = \mu N_B$. — (4)

Substituting equtn (4) in (3)

$$-P_a \cdot e + \frac{F_B}{\mu} \cdot m + F_B n = 0$$

$$F_B \left[\frac{m}{\mu} + n \right] = P_a \cdot e$$

$$F_B \left[m + \mu n \right] = P_a \cdot e \mu$$

$$\boxed{\frac{F_B}{P_a} = \frac{\mu e}{m + \mu n}}$$

For right side shoe i.e shoe A, it is called as leading shoe. The moment produced ~~as~~ by the friction force on the shoe, acts to rotate it against the drum. and increases the friction force devloped.

This "self - servo" action ~~developed~~ yields a mechanical advantage called as "brake factor".

So for leading shoe.

$$\boxed{\frac{F_B}{P_a} = \frac{\mu e}{m - n\mu}}$$

The brake is not only propotional to μ but if μ becomes so large than the denominator term will be small & may tend to 0.

In such case brake factor goes to infinity and brake will lock on application. (46)

For shoe B, i.e trailing shoe, the friction force acts to reduce the application force. The brake factor is much lower and higher forces are required to achieve desired braking torque.

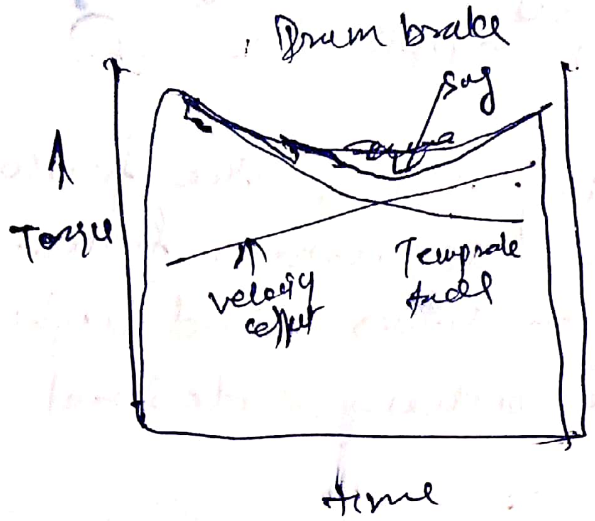
- By using 2 leading shoe, 2 trailing shoe or one of each different brake factor can be ~~opt~~ obtained.

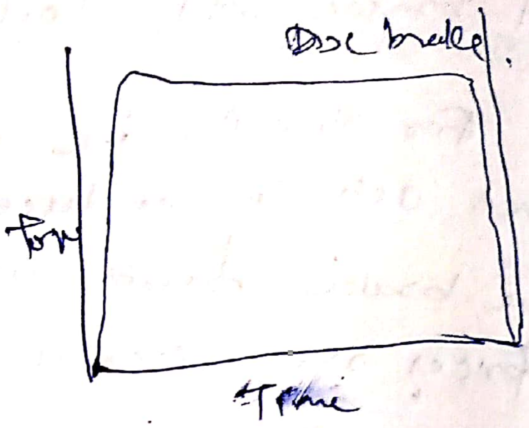
- The two servo brake has 2 leading shoe coupled to obtained a very high braking factor.

- high Brake factor → more noise.

~~if we change we~~
due to heating, wear & other factor ~~ca~~ will changes.

- Since disc Brake lack this self actuation effect and they have better torque consistency, but actuation force is high





Ex. 1) Pg-57.

$m = 1648\ kg$

$v_0 = 96.56\ km/h$

$F_b = 907\ kg$

$D_x = ?$

$S.D = ?$

$t_s = ?$

$E = ?$

Ans

1) $D_x = \frac{F_b}{m} = \frac{907 \times 9.81}{1648} = 5.39\ m/s^2$

2) $S.D = \frac{V_0^2}{2D_x} = \frac{[96.56 \times 10^3/3600]^2}{2(5.39)} = 66.71\ \cancel{71.22}\ m$

3) $t_s = \frac{V_0}{D_x} = \frac{(96.56 \times 10^3/3600)}{5.39} = 4.97\ sec$

4) $Energy = \frac{1}{2} m (v_0^2 - v_f^2)$

$= \frac{1}{2}(1648)\left[\left(\frac{96.56 \times 10^3}{3600}\right)^2 - 0\right]$

$$\boxed{E = 592811.64\ J.}$$

5) Initial Power = Force × velocity

$= F_b \times V_0 = (907 \times 9.81)\left(\frac{96.56 \times 10^3}{3600}\right) = 238655.28\ W$

1 HP = 746 W

Power (HP) = $238655.28 \times \frac{1}{746} = 319.91\ hp$

6) Average Power $= \frac{Energy}{t_s} = \frac{m}{2}\frac{[v_0^2]}{t_s} = \frac{1648}{2} \times \frac{\left(\frac{96.56 \times 10^3}{3600}\right)^2}{4.97}$

$= 160\ HP$

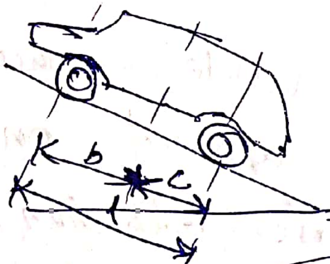
Brake Balance :-

Braking of a vehicle occurs ~~on~~ at ground level. The weight & K.E of vehicle acts through C.G. Due to this the vehicle pitches forward ~~on brakes~~ when brakes applied.

Because of Brakes, vehicle weight is effectively transfered from the rear wheel to front wheels. Hence front brake must absorb more K.E. than the rear brakes.

~~The maximum transfer of wt are is,~~

$$W_t = \mu \cdot \frac{Wh(-a_x)}{gl}$$



when we consider the deceleration of vehicle,

$$W_t = \mu \cdot \left(\frac{Wh}{gl}\right)(-a_x)$$

Note: The dynamic force shown is already derived in dynamic axle loading for case low speed & level ground.

$$\boxed{W_t = \mu \cdot \left(\frac{Wh}{gl}\right) \cdot (D_x)}$$

This wt is {Static wt. + Braking Load} on front wheel.

and wt is {Static wt. − Braking load} at Rear wheel.

But in acceleration the case is opposite.

Generally,

$W_f = 55\%$ of vehicle wt

→ front brake are designed to absorb extra of brake effort by selecting shoe drum or shoe disc combination type, brake size, μ of lining, wheel cylinder size & hydraulic pressures.

→ when full braking it is desirable to have the front brake lock up slightly ahead of the rear brakes. This causes the car to go straight ahead & not to spin out.

Qu. A vehicle has wheel base $l = 3h$. If brake is applied on all 4 wheels over a road whose adhesion factor is 0.6, determine the weight transfered from rear to front wheels.

Ans

$l = 3h$

$\mu = 0.6$

If brake is applied on all 4 wheels then, $D_x = \mu g$.

Brake applied 4-wheels

decelerating force

$F = m.(D_x)$

$F = \frac{W}{g}(D_x)$

But decelerating force is friction force. i.e

$F = \mu N$

$F = \mu W$

so, $\boxed{\mu = \frac{F}{W}}$

so, $\boxed{\frac{F}{W} = \frac{D_x}{g}}$

Comparing both

$\boxed{D_x = \mu g}$

weight Transfer,

$$W_T = \mu \cdot \left(\frac{Wh}{gl}\right) D_x$$

$$= (0.6)\left(\frac{W \cdot h}{g \cdot 3h}\right)(D_x)$$

$$= (\cancel{0.6})^{0.2}\left(\frac{W \cdot \cancel{h}}{3g\cancel{h}}\right)(\mu g)$$

$$\boxed{W_T = (0.2)\, W \cdot \mu}$$

% of weight transfer $= \frac{W_t}{W} \times 100$

$= \frac{0.2 W \mu}{\cancel{W}} \times 100 = 12\%$

Calculate the minimum stopping distance for a vehicle travelling at 60 km/h with Deceleration equal to the acceleration due to gravity. (48)

Ans.

$V_0 = 60\ km/h$

$D_x = g$

$S = \frac{V_0^2}{2D_x}$

$= \left[\frac{60 \times 10^3}{60 \times 60 \times \cancel{9.81}}\right]^2 \times \frac{1}{2g}$

$\boxed{S = 14.4\ m}$

Brake Torque

The braking torque is the twisting action caused by the drum or disc on the shoe during application of brakes.

Braking torque = (Effective axle ht.) × (Stopping force between tyre & road surface)

Front wheels → The torque absorbed by knuckle & suspension control arm

Rear wheels → Torque is absorbed by axle housing & leaf spring.

During emergency,

(Brake torque) >>> (Accelerating torque)

Brake supporting member must have sufficient strength.

Note:- ~~Ennergy~~ Emergency brake must hold the automobile on 30% slope indenfinetly after brake is applied.

Que. A car of mass 800kg is travelling at 36 km/h

Determine

(a) K.E of car

(b) Average braking force to bring it to rest in 20 m.

Ans (a) $V_0 = \frac{36 \times 1000}{60 \times 60}$

$\boxed{V_0 = 10 \text{ m/sec}}$

$K.E = \frac{1}{2} m V_0^2$

$= \frac{1}{2} \times 800 \times (10)^2 = 40 kJ$

$\boxed{K.E = 40 kJ}$

(b) Work done to stop Car = Change in K.E of Car.

(Force) (distance) = 40×10^3

$F \times 20 = 40 \times 10^3$

$F = 2 \times 10^3$

$\boxed{F = 2000 N}$

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* Braking Efficiency

The braking efficiency of a vehicle is defined as the braking force produced as a percentage of the total weight of vehicle.

$$\text{Braking efficiency}, \eta = \left(\frac{\text{Braking force}}{\text{weight of vehicle}}\right) \times 100$$

generally,

$\eta < 100\%$ because of insufficient road adhesion, vehicle on down gradient, ineffective braking system.

– Braking efficiency is similar to coefficient of friction. i.e. $\mu = \frac{\text{Frictional force}}{\text{normal load}}$.

So,

$$\mu = \frac{F}{R}$$

and

$$\eta = \frac{F}{R}$$

So, $\boxed{\mu = \eta}$

Thus a braking efficiency of 100% is equal to coefficient of friction of 1.

$$\eta(100\%) = \frac{F}{R} = \mu = 1$$

Brake efficiency can be derived from K.E possessed by vehicle. and work done to stop the vehicle.

We know,

$$(\text{force} \times \text{distance}) = \frac{1}{2} M \cdot V_0^2$$

But $M = \frac{W}{g}$

$$F \times S = \frac{W}{2g} V_0^2$$

$$S = \frac{W V_0^2}{2gF}$$

and $\frac{F}{W} = \mu = \eta$

$$S = \frac{V_0^2}{2g\eta}$$

or

$$\cancel{\eta = \frac{2gS}{V_0^2}} \qquad \boxed{\eta = \frac{V_0^2}{2gS} \times 100}$$

Que Determine the brake efficiency of a vehicle if the brakes bring the vehicle to rest in a distance of 15 m. from 60 km/hr.

$V_0 = 60$ km/hr

$= \frac{60 \times 10^3}{3600}$

$\boxed{V_0 = 16.6 \text{ m/s}}$

$$\eta = \frac{V_0^2}{2gS} \times 100$$

$$\eta = \left[\frac{(16.6)^2}{2 \times 9.81 \times 15}\right] \times 100$$

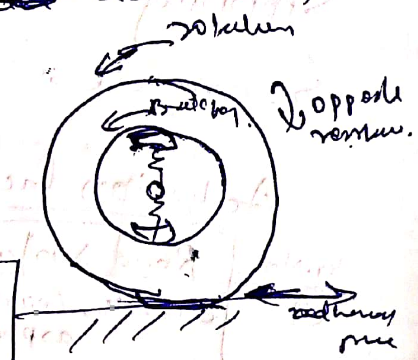
$\boxed{\eta = 94.38\ \%}$

Tyre Adhesion

The amount of force ~~general~~ applied on a shoe against a drum controls the resistance to rotation of road wheel.

Simultaneously the road surface has to drive the wheel around.

This driving force attains a limit ~~when~~ its limit when the resistance offered by the brake equals the maximum friction force between tyre and road which is known as ~~adh~~ adhesive force



The adhesion force is given by,

$$\boxed{\text{Adhesive force} = \text{load on wheel} \times \text{Coeff. of friction}}$$

When limit is reach, wheels starts to skid. & any extra force on brake shoe does not increase in rate of slowing down the vehicle, no matter how good is braking system.

hence adhesion between tyre & road is the governing factor.. for minimum stopping distance.

→ Road adhesion depends on :-

- (1) Type of road surface
- (2) Condition of surface e.g. wet, dry, icy, greasy etc.
- (3) Design of tire tread, mtrl. of tire, depth of tread.

The relationship between the decelerating force and the vertical load on a wheel is known as adhesion factor.

This factor is very much similar to coeff. of friction.

In ideal situation of braking, the wheel should always rotate till the point of stopping to ~~obtra~~ obtain greatest retarding resistance.

Adhesion factor

Sr. No	Road surface	Adhesion factor
1	Concrete, coarse asphalt dry	0.8
2	Tarmac, gritted bitumen dry	0.6
3	Concrete, coarse asphalt wet	0.5
4	Tarmac wet	0.4
5	Gritted bitumen tarmac wet	0.3
6	Gritted bitumen tarmac greasy	0.25
7	Gritted bitumen, snow compressed dry	0.2
8	Gritted bitumen snow compressed wet	0.15
9	Ice wet	0.1

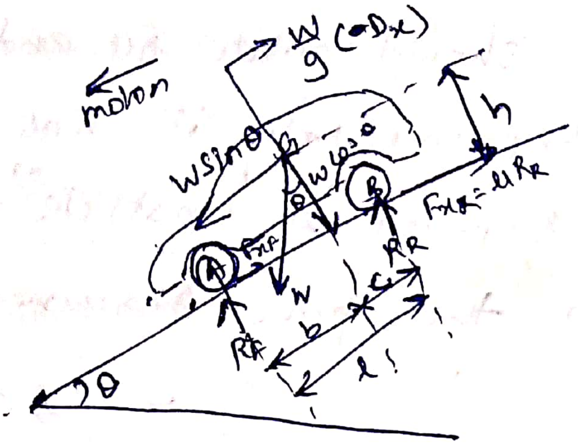
The vehicle should rotate till end & not lock because experimentally it has been confirmed that force required to "unstick" a tyre is greater than the force required to skid over surface.

51

$$\left(\text{force required to unstick tyre}\right) > \left(\text{Force required to skid over the surface}\right)$$

A wheel held on verge of skid not only provides the shortest distance, but also allows driver to maintain directional control of the vehicle.

* Braking of vehicle



$W \to$ wt. of vehicle
$R_F, R_R =$ Normal Reaction on wheels.
$\mu =$ coeff of friction

(a) Brakes applied on Rear wheel

$$\Sigma F_y = 0 = R_F + R_R - W\cos\theta$$

$$R_F + R_R = W\cos\theta \quad \text{—— (1)}$$

$$\Sigma F_x = 0 = F_{xR} + F_{xF} - W\sin\theta + \frac{W}{g}(a_x)$$

$$F_{xF} + F_{xR} = W\sin\theta + \frac{W}{g}(a_x)$$

$\therefore F_{xF} = 0$. (only Rear wheel Brake is applied)

$$F_{xR} = W\sin\theta + \frac{W}{g}(a_x) \quad \text{—— (2)}$$

~~Taking moment about G.~~

~~$\therefore R_F \neq 0, \; F_{xF} = 0$~~

~~so $\Sigma M_G = 0 = (R_R \cdot c) + (F_{xR} \cdot h)$~~

Taking moment at A.

$$\frac{W}{g}(-a_x)\cdot h - (R_R \cdot l) - (W\sin\theta)h + W\cos\theta \cdot b = 0$$

$$\frac{W}{g}a_x \cdot h + R_R \cdot l + Wh\sin\theta = Wb\cos\theta \quad \text{—— (3)}$$

Substituting from equation (2) i.e.

$$F_{xR} = \mu R_R = W\sin\theta + \frac{W}{g}(a_x)$$

$$R_R = \left(\frac{W}{\mu}\right)\sin\theta + \left(\frac{W}{g\mu}\right)(a_x) \quad \text{—— (2a)}$$

in equation (3).

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$$\frac{W}{g}a_x h + \frac{Wl}{\mu}\sin\theta + \frac{Wl}{g\mu}a_x + Wh\sin\theta = Wb\cos\theta$$ (52)

$$\cancel{W}\left[\frac{a_x h}{g} + \frac{l\sin\theta}{\mu} + \frac{l a_x}{g\mu} + h\sin\theta\right] = \cancel{W}b\cos\theta$$

$$\frac{a_x}{g}\left[h + \frac{l}{\mu}\right] + \sin\theta\left[h + \frac{l}{\mu}\right] = b\cos\theta$$

$$\cancel{\frac{a_x}{g} = b\cos\theta - \left[h+\frac{l}{\mu}\right]\frac{g x}{g}}$$

$$\frac{a_x}{g}\left(h + \frac{l}{\mu}\right] = b\cos\theta - \left(h+\frac{l}{\mu}\right)\sin\theta$$

$$\frac{a_x}{g} = \frac{b\cos\theta}{\left(h+\frac{l}{\mu}\right)} - \frac{\cancel{\left(h+\frac{l}{\mu}\right)}\sin\theta}{\cancel{\left(h+\frac{l}{\mu}\right)}}$$

$$\boxed{\frac{a_x}{g} = \frac{b\cos\theta}{\left(h+\frac{l}{\mu}\right)} - \sin\theta} \quad \text{— (4)}$$

Substituting equation (4) in equation (2)

i.e

$$R_R = \left(\frac{W}{\mu}\right)\sin\theta + \left(\frac{W}{\mu}\right)\left(\frac{a_x}{g}\right)$$

$$R_R = \left(\frac{W}{\mu}\right)\sin\theta + \left(\frac{W}{\mu}\right)\left[\frac{b\cos\theta}{h+\frac{l}{\mu}} - \sin\theta\right]$$

$$R_R = \cancel{\left(\frac{W}{\mu}\right)\sin\theta} + \frac{Wb\cos\theta}{\mu\left(h+\frac{l}{\mu}\right)} - \cancel{\frac{W}{\mu}\sin\theta}$$

$$\boxed{R_R = \frac{Wb\cos\theta}{h\mu + l}}$$

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and therefore substituting R_R in equation ①

$$R_F = W\cos\theta - R_R$$

$$= W\cos\theta - \left(\frac{Wb\cos\theta}{h\mu + l}\right)$$

$$= W\cos\theta\left[1 - \frac{b}{(h\mu + l)}\right]$$

$$\boxed{R_F = W\cos\theta\left(\frac{h\mu + l - b}{h\mu + l}\right)}$$

(b) Brakes applied to front wheels.

$$\Sigma F_x = 0 = R_F + R_R = W\cos\theta \quad \text{——— ①}$$

$$\Sigma F_y = 0 = F_{xF} + F_{xR} - W\sin\theta + \frac{W}{g}(-a_x)$$

$$0 = F_{xF} + F_{xR} - W\sin\theta - \frac{W}{g}a_x$$

$\therefore F_{xR} = 0$ (only front brakes applied)

$$0 = F_{xF} - W\sin\theta - \frac{W}{g}a_x$$

$$F_{xF} = W\sin\theta + \frac{W}{g}a_x$$

$$\mu R_F = W\sin\theta + \frac{W}{g}a_x$$

$$R_F = \left(\frac{W}{\mu}\right)\sin\theta + \left(\frac{W}{\mu}\right)\frac{a_x}{g} \quad \text{——— ②}$$

$$\Sigma M_B = 0 = \left\{\frac{W}{g}(-a_x)\cdot h\right\} - \left\{W\cos\theta\cdot c\right\} - \left\{W\sin\theta\cdot h\right\} + (R_F\cdot l) \quad \text{——— ③}$$

Substituting equn (2) in (3)

$$0 = -\frac{W}{g}a_x \cdot h - WC\cos\theta - Wh\sin\theta + \frac{Wl}{\mu}\sin\theta + \frac{Wl}{\mu}\frac{a_x}{g} = 0$$

$$\frac{W}{g}a_x h + WC\cos\theta + Wh\sin\theta = \frac{Wl}{\mu}\sin\theta + \frac{Wl}{\mu}\frac{a_x}{g}$$

$$\frac{a_x}{g}h + c\cos\theta + h\sin\theta = \left(\frac{l}{\mu}\right)\sin\theta + \left(\frac{l}{\mu}\right)\frac{a_x}{g}$$

$$c\cos\theta - \left(\frac{l}{\mu} - h\right)\sin\theta = \frac{a_x}{g}\left(\frac{l}{\mu} - h\right)$$

$$\frac{a_x}{g} = \frac{C \cdot \cos\theta}{\left(\frac{l}{\mu} - h\right)} - \sin\theta$$

$$\boxed{\left(\frac{a_x}{g}\right) = \frac{\mu C\cos\theta}{l - h\mu} - \sin\theta} \quad \text{--- (4)}$$

Substituting above equn (4) in equtn (2)

$$R_f = \left(\frac{W}{\mu}\right)\sin\theta + \left(\frac{W}{\mu}\right)\left\{\frac{\mu C\cos\theta}{l - h\mu} - \sin\theta\right\}$$

$$= \cancel{\frac{W}{\mu}\sin\theta} + \frac{WC\cos\theta}{l - h\mu} - \cancel{\frac{W}{\mu}\sin\theta}$$

$$\boxed{R_f = \frac{WC\cos\theta}{l - h\mu}}$$

and substituting in equtn (1)

$$R_R = W\cos\theta - R_f$$

$$= W\cos\theta - \frac{W \cdot C\cos\theta}{l - h\mu}$$

$$= W\cos\theta\left[1 - \frac{C}{(l - h\mu)}\right]$$

$$\boxed{R_R = W_r \cos\theta \left\{ \frac{l - h\mu - c}{l - h\mu} \right\}}$$

(c) Brakes applied to all 4 wheels

In this case both F_{xR} & F_{xF} acts at rear & front wheels ~~respectiv~~ respectively giving maximum possible braking force.

$\Sigma F_x = 0$,

$$F_{xR} + F_{xF} = W\sin\theta - \frac{W}{g}(D_x)$$

$$F_{xR} + F_{xF} = W\sin\theta + \frac{W}{g}a_x$$

$$\mu R_R + \mu R_F = W\sin\theta + \frac{W}{g}a_x \quad \text{--- (1)}$$

$\Sigma F_y = 0$,

$$R_F + R_R = W\cos\theta \quad \text{--- (2)}.$$

Solving equtn (1) & (2).

$$\mu W\cos\theta = W\sin\theta + \frac{W}{g}a_x$$

$$\boxed{\frac{a_x}{g} = \mu\cos\theta - \sin\theta} \quad \text{--- (3)}$$

~~Substituting eqation~~

Taking moment at B,

$$\Sigma M_B = 0 = \left\{\frac{W}{g}(-a_x).h\right\} - \left\{W\cos\theta \cdot c\right\} - \left\{Wh\sin\theta\right\} + R_F \cdot l = 0$$

$$R_F \cdot l = \left\{W \cdot c\cos\theta\right\} + \left\{Wh\sin\theta\right\} + \left(\frac{Wh}{g}a_x\right)$$

Substitue equtn (3) in above

$$R_F = \frac{1}{l}\left[W \cdot c\cos\theta + Wh\sin\theta + W \cdot h(\mu\cos\theta - \sin\theta)\right]$$

$$= \frac{1}{l}\left[Wc\cos\theta + \cancel{Wh\sin\theta} + \mu Wh\cos\theta - \cancel{Wh\sin\theta}\right]$$

(54)

$$\boxed{R_F = \frac{W\cos\theta}{l}\left[c + \mu h\right]}$$

Similarly Substituting R_F in equation (2)

$$R_R = W\cos\theta - R_F$$

$$= W\cos\theta - \frac{W\cos\theta}{l}\left[c + \mu h\right]$$

$$= W\cos\theta\left[1 - \frac{(c + \mu h)}{l}\right]$$

$$\boxed{R_R = \frac{W\cos\theta}{l}\left[l - c - \mu h\right]}$$

In case vehicle moves on level road then $\theta = 0$ & hence in above 9 expression.

$\sin\theta = 0$, $\cos\theta = 1$.

Q. A motor car has a wheel base of 2.64 m, height of C.G. above ground is 0.61 m & it is 1.12 m in front of rear axle. If the car is travelling at 40 km/hr on a level track, determine the minimum distance in which the car may be stopped

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when,

(a) Rear wheels are braked

(b) front wheels are braked

(c) All wheels are braked

μ between tyre & road is 0.6

Ans

$l = 2.64 m$

$h = 0.61 m$

$c = 1.12 m$

$v_0 = 40 km/hr = \frac{40}{3.6} = 11.11 m/s.$

$\theta = 0$ (level road)

$S = ?$

(a) when rear wheels are braked.

$$\frac{a_x}{g} = \frac{b\cos\theta}{\left(h + \frac{l}{\mu}\right)} - \sin\theta$$

$$\frac{a_x}{g} = \frac{b}{\left(h + \frac{l}{\mu}\right)} - 0$$

$$\frac{a_x}{g} = \cancel{\frac{b - \left(h + \frac{l}{\mu}\right)}{h + \frac{l}{\mu}}} \quad \frac{b}{\left(h + \frac{l}{\mu}\right)}$$

$$\frac{a_x}{g} = \frac{b\mu}{h\mu + l}$$

$$\frac{a_x}{g} = \frac{(l - c)\mu}{(l + \mu h)} = \frac{(2.64 - 1.12)\,0.6}{\{2.64 + (0.6 \times 0.61)\}}$$

$$\boxed{a_x = 2.98 \ m/s^2}$$

$$S = \frac{V_0^2}{2a_x}$$

$$= \frac{(11.11)^2}{2 \times 2.98} \approx 20.55m$$

$$\boxed{S = 20.55m}$$

(b) when Front wheels are braked

$$\frac{a_x}{g} = \frac{C \cdot \cos\theta}{\left(\frac{l}{\mu} - h\right)} - \sin\theta$$

$$\frac{a_x}{g} = \frac{C \cdot \mu}{(l - \mu h)}$$

$$a_x = \frac{g \mu \cdot C}{l - \mu h}$$

$$a_x = \frac{9.81 \times 0.6 \times 1.12}{2.64 - [0.6 \times 0.61]}$$

$$\boxed{a_x = 2.9 m/s^2}$$

$$S = \frac{V_0^2}{2a_x}$$

$$= \frac{(11.1)^2}{2(2.9)}$$

$$\boxed{S = 21.25m}$$

(c) when all Brakes are applied.

$$\frac{a_x}{g} = \mu \cos\theta - \sin\theta$$

$$\frac{a_x}{g} = \mu.$$

$a_x = 0.6 \cdot 9.81$

$$\boxed{a_x = 5.886 \, m/s^2}$$

$$S = \frac{V_0^2}{2a_x}$$

$$S = \frac{123.21}{2 \times 5.886}$$

$$\boxed{S = 10.5 \, m}$$

Que. A motor car weights 13341.5 N and has a wheelbase of 2.65 m. The C.G is 1.27 m behind the front axle & 0.76 m above the ground level. Maximum braking on all 4 wheels on level ground will bring the vehicle uniformly to rest from a speed of 64 Km/hr in a distance of 25.9 m. Calculate the value of adhesion between tyre & the road. Under the same road condition, the vehicle descends a hill of gradient 1 in 20 and is braked on front wheels only. Determine the load distribution between the front & rear wheels and the distance required to bring the car to rest.

Ans,

$W = 13341.5 N$

$\lambda = 2.65 m$

$b = 1.27 m$

$h = 0.76 m$

$V_0 = 64 \, km/hr$, $S = 25.9 m$ { brake applied on all 4 wheels

$a_x = \frac{V_0^2}{2S}$

$= \frac{\left(\frac{64}{3.6}\right)^2}{2 \times 25.9}$

$\boxed{a_x = 6.1\, m/s^2}$

when brake applied on all 4 wheels

$\mu = \frac{a_x}{g}$

$= \frac{6.1}{9.81}$

$\boxed{\mu = 0.622}$

(b) when vehicle is on hill gradient of 1 in 20, & in front wheel braked,

$\tan\theta = \frac{1}{20} = 0.05 \approx \sin\theta$

$\cos\theta \approx 0.999 = 1$.

$\frac{a_x}{g} = \frac{c\cos\theta}{\left(\frac{l}{\mu} - h\right)} - \sin\theta$

$= \frac{c\mu\cos\theta}{(l - h\mu)} - \sin\theta$

$a_x = \frac{(l-b) \cdot 0.622\,(1) \times 9.81}{2.65 - (0.76 \times 0.62)} - (0.05 \times 9.81)$

$\boxed{a_x = 3.38\, m/s^2}$

$$S = \frac{v_0^2}{2a_x}$$

$$= \frac{\left(\frac{64}{3.6}\right)^2}{2 \times 3.38}$$

$$\boxed{S = 46.8\,m}$$

Distribution of load on front wheels,

$$R_F = \frac{W \cdot C \cos\theta}{l - h\mu}$$

$$\boxed{R_F = 8457.2\,N}$$

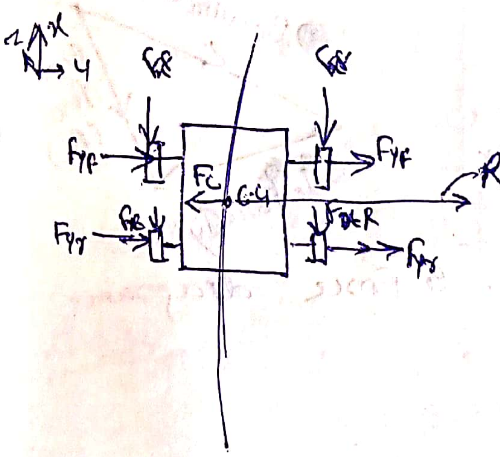
$$R_R = W\cos\theta \left[\frac{l - h\mu - C}{l - h\mu}\right]$$

$$\boxed{R_R = 4884.4\,N}$$

Braking of vehicle moving in a curved Path

- While moving outwards vehicle comes under the influence of centrifugal force.
- The centrifugal force tries to move vehicle outwards & this action is made futile by side forces acting on tyres in reverse direction of centrifugal force.

- When brake is applied while moving along a curved path, the frictional forces acting on tyre becomes more complex.



let W = wt. of vehicle in N

R_c = Radius of curved Path, m

V = Forward velocity of all parts. m/s

F_{yF}, F_{yr} = Side forces at front & rear wheels respectively, N

F_{xF}, F_{xr} = Braking forces at front & rear wheels respectively, N.

$$\text{Centrifugal Force, } F_c = \frac{WV^2}{R_c}, \text{N.}$$

This centrifugal force will act through C.G. in outward direction

As we can see the force diagram, we can say that the radius of curvature of path is very large compared to the dimensions of vehicle, so we can assume that F_{yF} and F_{yr} are parallel to each other.

For simplification, it is assumed that the forces ~~on~~ at the vehicle wheels are compressed into a single force on a single plane passing through C.G. and neglecting the rolling effect

lateral acclth

on the wheel reactions due to centrifugal action & turning tendency during braking caused by unequal forces at inner & outer wheels.

So,

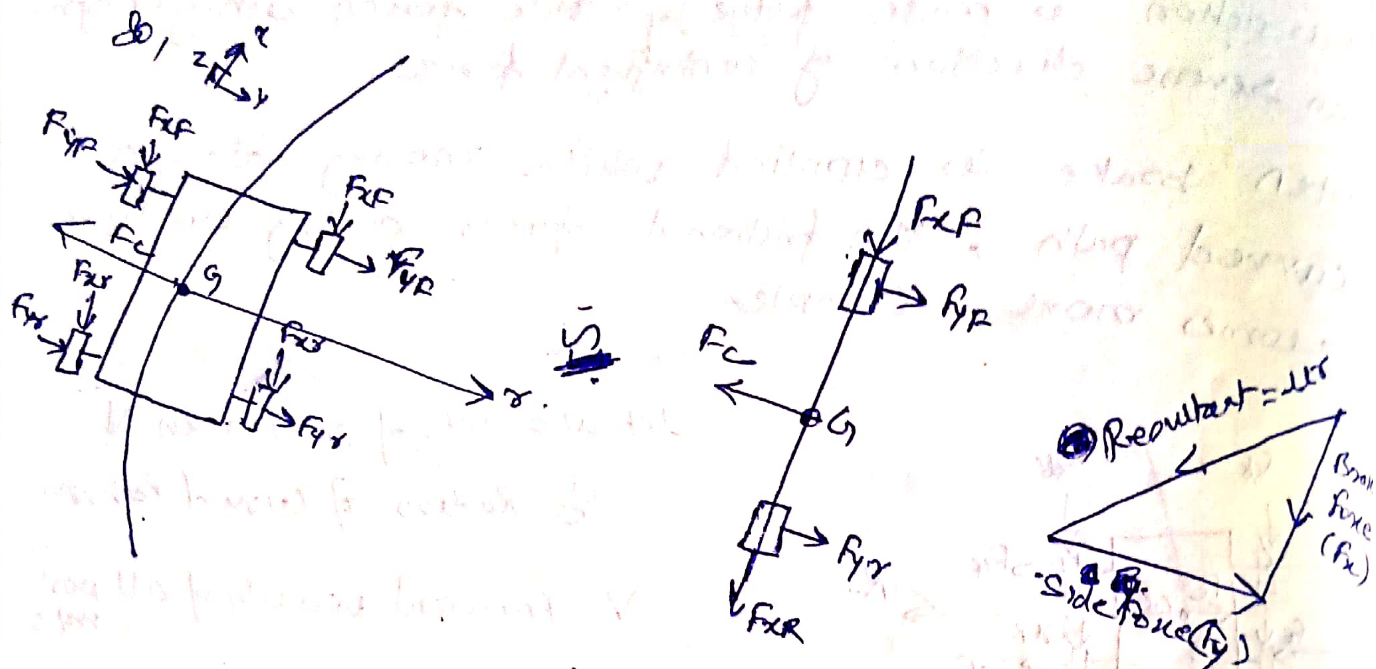


Fig:- Equivalent fig of vehicle. & Force diagram.

Referring the force triangle,

R = vertical load on the wheel

μ = coeff. of adhesion.

A part of frictional force μR resist the side slip and the rest is utilized for braking.

It is quite clear from force triangle that the braking capacity of a vehicle is reduced while moving along a curved path.

Finally it can be concluded that if the value of μ is very high then the vehicle moving about a certain speed may overturn before it slide side ways.

Qu A motorcycle has a wheelbase 1.44 m apart. The C.G. of the cycle and rider is 0.76 m above the ground level, and 0.61 m in front of the rear axle. The coeff. of friction between the tyres and the road is 0.75. The rear wheel is braked, find the greatest retardation that can be obtained.

(a) If the cycle is moving in a straight path.

(b) If it is going around a curve of 45.7 m radius at 48 km/h.

Assume ~~for~~ a level road & neglect air resistance. Neglect rotational inertia & obliquity when turning.

Ans: $l = 1.44 m$

$h = 0.76 m$

$C = 0.61 m$

$\mu = 0.75$ {rear wheels are braked}

$a_x = ?$

(a). If vehicle moving on a straight path.

$$a_x = g\left\{\frac{b\cos\theta}{h + \frac{l}{\mu}} - \sin\theta\right\} \qquad \therefore \theta = 0^\circ$$

$$a_x = g\left\{\frac{b\mu}{l + h\mu}\right\}$$

$$a_x = 9.81\left\{\frac{(l-c)\,0.75}{1.44 + (0.76 \times 0.75)}\right\}$$

$$a_x = 9.81\left\{\frac{(1.44 - 0.61)\,0.75}{(1.44) + (0.76 \times 0.75)}\right\} = 3.038\ m/s^2$$

Qu. A motorcycle has a wheelbase 1.44 m apart. The C.G. of the cycle and rider is 0.76 m above the ground level, and 0.61 m is front of the rear axle. The coeff. of friction between the tyres and the road is 0.75. The rear wheel is braked, find the greatest retardation that can be obtained. (58)

(a) If the cycle is moving in a straight path.

(b) If it is going around a curve of 45.7 m radius at 48 km/h.

Assume ~~for~~ a level road & neglect air resistance. Neglect rotational inertia & obliquity when turning.

Ans. $l = 1.44 m$

$h = 0.76 m$

$c = 0.61 m$

$\mu = 0.75$ {rear wheels are braked}

$a_x = ?$

(a). If vehicle moving on a straight path.

$$a_x = g\left\{\frac{b\cos\theta}{h + \frac{l}{\mu}} - \sin\theta\right\} \quad \therefore \theta = 0°$$

$$a_x = g\left\{\frac{b\mu}{l + h\mu}\right\}$$

$$a_x = 9.81\left\{\frac{(l-c)\,0.75}{1.44 + (0.76 \times 0.75)}\right\}$$

$$a_x = 9.81\left\{\frac{(1.44 - 0.61)\,0.75}{(1.44) + (0.76 \times 0.75)}\right\} = 3.038\ m/s^2$$

lateral accln →

$$\boxed{a_x = 3.038\ m/s^2}$$

(b) When vehicle moving on a curved path:

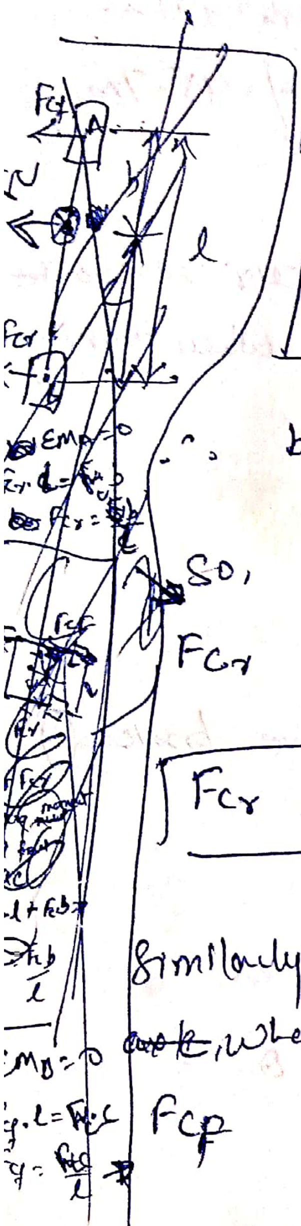
$R_c = 45.7\,m$

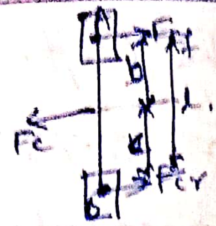
$V_0 = 48\ km/h$

$$F_c = \frac{W}{g}\frac{V_0^2}{R_c}$$

$$F_c = \frac{W}{9.81} \times \frac{\left(\frac{48}{3.6}\right)^2}{45.7}$$

$$\boxed{F_c = 0.397\,W}\ ,\ kgf$$





Taking moment

$F_{cr} \cdot l = F_c \cdot b$

$$\boxed{F_{cr} = \frac{F_c \cdot b}{l}}$$

$F_{cr} = 0.397W \ldots$

$$\boxed{F_{cr} = 0.2\ldots}$$

Taking moment at B

$F_{cf} \cdot l = F_c \cdot c$

$$\boxed{F_{cf} = \frac{F_c \cdot c}{l}}$$

$$F_{cf} = \frac{0.397 \times 0.61}{1.44} = 0.168\ldots$$

$\therefore\ b = l - c = 1.44 - 0.61 = 0.83\ m$

So, the centrifugal force shared by rear wheel.

$$F_{cr} = \frac{0.83}{1.44} \times 0.397W$$

$$\boxed{F_{cr} = 0.229\,W}\ ,\ kgf$$

Similarly centrifugal force shared by front wheels, $\therefore c = 0.61$

$$F_{cf} = \frac{0.61}{1.44} \times 0.397W$$

$$\boxed{F_{cf} = 0.168\,W}\ ,\ kgf$$

[The maximum frictional force at rear wheel] = [μR_R]

The part of μR_R is available for braking in direction of motion.

Scanned with CamScanner

$$(\mu R_R)^2 = F_x^2 + F_y^2$$

$$F_x^2 = F_y^2 - (\mu R_R)^2$$

$F_{cr} \cdot l + F_c \cdot b$

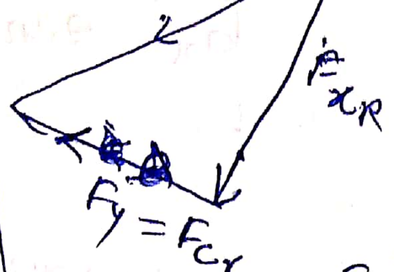
$F_{cr} = \frac{F_c \cdot b}{l}$

$$F_{x_R} = \sqrt{F_{c_r}^2 - (\mu R_R)^2}$$

$F_{cf} \cdot l + F_c \cdot l = 0$

$F_{cf} = \frac{F_c\, l}{l}$

$$\left(\frac{W}{g}\right) a_x = \cancel{F_{cr}^2 - (\mu R_R)^2}$$



$$F_{xR}^2 + F_{cr}^2 = (\mu R_R)^2$$

$$F_{xR}^2 = (\mu R_R)^2 - F_{cr}^2$$

Hence Braking force on rear wheels will be.

$$F_{xR} = \sqrt{(\mu R_R)^2 - (0.229W)^2}$$

$$\left[\left(\frac{W}{g}\right) a_x\right]^2 = (\mu R_R)^2 - (0.229W)^2$$

$$(\mu R_R)^2 = \left[\left(\frac{W}{g}\right) a_x\right]^2 + (0.229W)^2$$

$$R_R^2 = \left(\frac{W a_x}{g\mu}\right)^2 + \left(\frac{0.229W}{\mu}\right)^2$$

$$R_R^2 = (0.136 \times a_x W)^2 + (0.305W)^2 \text{ ——— (1)}$$

Taking moment of forces about the front wheel.

$$\frac{W}{g} a_x \cdot h + R_R \cdot l + Wh\sin\theta = Wb\cos\theta$$

$\therefore \theta = 0°$

$$\frac{W}{g} a_x \cdot h + R_R \cdot l = Wb$$

$$\left(\left(\frac{W}{g} a_x\right) \times 0.76\right) + (R_R \times 1.44) = (W \cdot 0.83)$$

$$R_R^2 = \left[\frac{0.83W}{1.44} - \frac{W}{g} a_x \left(\frac{0.76}{1.44}\right)\right]^2 \text{ ——— (2)}$$

Solving equations (1) & (2).

equating equation ① & ②

$$\boxed{a_x = 2.425\ m/s^2}$$

Now to solve eqution

equation ②

$$R_R^2 = (0.576\,W - 0.05275\,a_x W)^2$$

$$R_R^2 = (0.576\,W)^2 + (0.05275\,a_x W)^2 - 2 a_x W^2\, 0.052$$

and eqution ①

$$R_R^2 = (0.136\,a_x W)^2 + (0.305\,W)^2$$

equating both

$$(0.576\,W)^2 + (0.05275\,a_x W)^2 - 2(a_x W\, 0.05275)^2 = (0.136\,a_x W)^2 + (0.305\,W)^2$$

$$(0.576)^2 + (0.05275\,W)^2 - 2(a_x \times 0.05275)^2 = (0.136\,a_x)^2 + (0.305\,W)^2$$

$$\boxed{a_x = 2.425\ m/s^2}$$